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ABSTRACT

Designed to accompany the student text, this guide presents lessons relating specific mathematical concepts to the interdisciplinary biomedical curriculum. Each lesson includes discussion of objectives, recommended class periods involved in the lesson, and a section of overview and remarks. Advice on teaching the concept, integrating the lesson with other disciplines of the curriculum, and options for teaching the lesson are presented in the overview and remarks section. (RE)

BIOMEDICAL MATHEMATICS

UNIT IV

SYMBOLIC LOGIC, TRIGONOMETRY AND STATISTICS

INSTRUCTOR'S MANUAL
REVISED VERSION, 1976

THE BIOMEDICAL INTERDISCIPLINARY CURRICULUM PROJECT

SUPPORTED BY THE NATIONAL SCIENCE FOUNDATION

TABLE OF CONTENTS

LESSON 1: LOGICAL STATEMENTS

OBJECTIVES:

The student will:

- decide whether given sentences satisfy the definition of a logical statement.

- interconvert logical statements in sentence form and logical statements in symbolic form.

PERIODS RECOMMENDED: One

OVERVIEW AND REMARKS:

This lesson is the first in a sequence of nine lessons on symbolic logic and its application to the design of computer circuits. This sequence is intended to provide a theoretical foundation for the applications in Science Lessons 9 to 13. In the Science sequence the students will design circuits for diagnosing brain diseases. These circuits will indicate disease possibilities when a set of symptoms are introduced. The students will then construct the circuits on the BIP-powered Brain Frame by mounting component cards and making simple wire interconnections.

There are several ways in which this interdisciplinary sequence can be approached. We have suggested two options below. In choosing one of these, or in creating your own, we suggest that you keep the following considerations in mind.

First, the Mathematics sequence is mostly theoretical. Retention and motivation will both suffer if this sequence is not followed shortly by some sort of working experience with the Brain Frame. Such an experience could happen in either Mathematics or Science class.

Second, the Science sequence depends on the material presented in the Mathematics class. Therefore, the science teacher will need your support in becoming familiarized with the language of symbolic logic. The cooperation that you show will determine the smoothness of the Mathematics-Science transition and hence the quality of the interdisciplinary experience for the students. A day's team teaching at the beginning of the Science sequence would be a big help. Also you might devote a few minutes in subsequent Mathematics classes to clearing up points of confusion arising in Science class.

We suggest one of the following two options for handling the sequence, the choice depending on relative class schedules.

Option 1: In the event that the Mathematics and Science classes begin Unit IV at roughly the same time the sequences can be taught as written. The one constraint is that Mathematics Lesson 8 must be completed before Science Lesson 10 is presented. You should arrange to be present in the Science class when Activity 10 and Worksheet 10 are presented.

Option 2: In the event that the Mathematics class begins Unit IV sometime before the Science class, Science Worksheet 9 and all of Science Lesson 10 should be presented in Mathematics class immediately after completion of the logic sequence. Additional time could be spent constructing circuits appearing in the Mathematics Text, or circuits that relate to simple applications (e.g., you could consider a situation in which three people are voting and you want a circuit which will turn on a light for a majority of yes votes). The computer-diagnosis Science material can be covered whenever it is reached in the Science schedule. At that time, a review in the Mathematics class would be essential.

Turning to the Mathematics sequence itself, this first lesson is concerned with logical statements. A logical statement is simply a clearly formed, unambiguous sentence. Computer diagnosis involving making decisions about the truth of statements, which is impossible if they are not clearly formulated. The examples of the text could be supplemented by having students construct logical statements based on a page of a newspaper. After the symbolic logic symbols are introduced, the students

may have trouble keeping the symbols "$\wedge$" (and) and "$\vee$" (or) straight. You might point out that "$\wedge$" looks like the letter "A" which is the first letter in "and."

KEY--PROBLEM SET 1:

1. S	5. S	9. S	13. $p \wedge q$	17. $p \wedge q \wedge r$
2. NS	6. NS	10. S	14. $p \wedge \overline{p}$	18. $p \vee \overline{q} \vee \overline{r}$
3. NS	7. S	11. S	15. $q \vee \overline{q}$	19. $p \wedge (q \vee r)$
4. S	8. S	12. $p \vee r$	16. $\overline{p} \vee \overline{q}$	20. $\overline{q \vee r}$
				21. $\overline{p} \vee (\overline{q \wedge r})$

22. Strephon has strep or John has jaundice.
23. Rick does not have rickets.
24. John does not have jaundice and Rick has rickets.
25. Strephon does not have strep and John has jaundice and Rick does not have rickets.
26. Strephon has strep and John does not have jaundice, or Rick has rickets.

Another possible answer is:

At least one of the following holds.

a. Strephon has strep and John does not have jaundice.

b. Rick has rickets.

27. Strephon has strep or John does not have jaundice, and Strephon does not have strep or Rick has rickets.

Another possible answer is:

Both the following are true.

a. Strephon has strep or John does not have jaundice.

b. Strephon does not have strep or Rick has rickets.

LESSON 2: TRUTH TABLES

OBJECTIVE:

The student will construct truth tables for given symbolic statements.

PERIODS RECOMMENDED: One or two.

OVERVIEW AND REMARKS:

At the end of the problem set are several logic puzzles which can be solved by the use of truth tables. Subsequent material does not depend on completion of these problems. The problems are more challenging than the average, and the level of your class will dictate how they can best be used, if at all.

KEY--PROBLEM SET 2:

1. true	6. true	11. true, true
2. false	7. false	12. true
3. false	8. false	13. false, false, false
4. false	9. true	
5. false	10. false, false	

14.

p	q	$\bar{p}$	$\bar{p}\wedge q$
1	1	0	0
1	0	0	0
0	1	1	1
0	0	1	0

15.

p	q	$\bar{p}$	$\bar{q}$	$\bar{p}\vee\bar{q}$
1	1	0	0	0
1	0	0	1	1
0	1	1	0	1
0	0	1	1	1

16.

p	q	$p\wedge q$	$\overline{p\wedge q}$
1	1	1	0
1	0	0	1
0	1	0	1
0	0	0	1

17.

p	q	r	$q\wedge r$	$p\vee(q\wedge r)$
1	1	1	1	1
1	0	1	0	1
0	1	1	1	1
0	0	1	0	0
1	1	0	0	1
1	0	0	0	1
0	1	0	0	0
0	0	0	0	0

18.

p	q	$\bar{q}$	$p\wedge\bar{q}$
1	1	0	0
1	0	1	1
0	1	0	0
0	0	1	0

19.

p	q	$\bar{p}$	$\bar{p}\vee q$
1	1	0	1
1	0	0	0
0	1	1	1
0	0	1	1

20.

p	q	$\bar{p}$	$\bar{p}\vee q$	$\overline{\bar{p}\vee q}$
1	1	0	1	0
1	0	0	0	1
0	1	1	1	0
0	0	1	1	0

21.

	Clark	Brown	Jones	Smith
Acupuncturist	0	0	1	0
Brain Surgeon	0	0	0	1
Dermatologist	1	0	0	0
Psychiatrist	0	1	0	0

22. surgeon - Stitch
radiologist - Roentgen
anesthesiologist - Miasma

23. Bed #1 - Snerd
#2 - Twang
#3 - Burke
#4 - Fosdick
#5 - Ascot

24. pitcher - Harry
catcher - Allen
first base - Paul
second base - Jerry
third base - Andy
shortstop - Ed
left field - Sam
center field - Bill
right field - Mike

LESSON 3: EQUIVALENT STATEMENTS

OBJECTIVE:

The student will use truth tables to decide when two symbolic statements are equivalent.

PERIODS RECOMMENDED:

One

KEY--PROBLEM SET 3

1. E 3. D 5. C

2. A 4. B 6. $\overline{s \vee t} = \bar{s} \wedge \bar{t}$

7. a.

p	q	$\bar{q}$	$p \wedge q$	$p \wedge \bar{q}$	$(p \wedge q) \vee (p \wedge \bar{q})$
1	1	0	1	0	1
1	0	1	0	1	1
0	1	0	0	0	0
0	0	1	0	0	0

b. $(p \wedge q) \vee (p \wedge \bar{q}) = p$

8. a.

r	s	$\bar{r}$	$r \vee s$	$\bar{r} \vee s$	$(r \vee s) \wedge (\bar{r} \vee s)$
1	1	0	1	1	1
1	0	0	1	0	0
0	1	1	1	1	1
0	0	1	0	1	0

b. $(r \vee s) \wedge (\bar{r} \vee s) = s$

9. a.

p	q	$\bar{p}$	$\bar{q}$	$\bar{p} \vee q$	$p \wedge \bar{q}$	$\overline{p \wedge \bar{q}}$
1	1	0	0	1	0	1
1	0	0	1	0	1	0
0	1	1	0	1	0	1
0	0	1	1	1	0	1

b. $\overline{p \wedge \bar{q}} = \bar{p} \vee q$

10. a.

s	t	$\bar{s}$	$\bar{t}$	$s \wedge \bar{t}$	$\bar{s} \vee t$	$\overline{\bar{s} \vee t}$
1	1	0	0	0	1	0
1	0	0	1	1	0	1
0	1	1	0	0	1	0
0	0	1	1	0	1	0

b. $\overline{\bar{s} \vee t} = s \wedge \bar{t}$

11. a.

s	t	$\bar{s}$	$\bar{t}$	$s \vee t$	$\bar{s} \wedge \bar{t}$	$\overline{\bar{s} \wedge \bar{t}}$
1	1	0	0	1	0	1
1	0	0	1	1	0	1
0	1	1	0	1	0	1
0	0	1	1	0	1	0

b. $\overline{\bar{s} \wedge \bar{t}} = s \vee t$

LESSON 4: SUBSTITUTION AND DE MORGAN'S LAWS

OBJECTIVES:

The student will:

- convert a given equivalence into a new equivalence through the process of substitution.
- use De Morgan's Laws to simplify statements in symbolic form.

PERIODS RECOMMENDED:

One

KEY--PROBLEM SET 4:

1. b.
2. c.
3. b.
4. $\overline{s \vee q} = \bar{s} \wedge \bar{q}$
5. $\overline{p \wedge \bar{w}} = \bar{p} \vee \bar{\bar{w}}$
6. $(p \vee q) \wedge \bar{\bar{t}} = (p \vee q) \wedge t$
7. $\overline{w \vee (r \wedge s)} = \bar{w} \wedge \overline{(r \wedge s)}$
8. $\overline{(r \wedge s) \vee \bar{s}} = (\overline{r \wedge s}) \wedge \bar{\bar{s}}$
9. $\overline{\bar{r} \wedge (r \vee \bar{s})} = \bar{\bar{r}} \vee \overline{(r \vee \bar{s})}$
10. $\bar{\bar{s}} = s$
11. $\overline{r \wedge \bar{s}} = \bar{r} \vee \bar{\bar{s}} = \bar{r} \vee s$
12. $\overline{\bar{p} \wedge r} = \bar{\bar{p}} \vee \bar{r} = p \vee \bar{r}$
13. $\overline{\bar{s} \wedge \bar{t}} = \bar{\bar{s}} \vee \bar{\bar{t}} = s \vee t$
14. $\overline{\bar{q} \vee r} = \bar{\bar{q}} \wedge \bar{r} = q \wedge \bar{r}$
15. $\overline{\bar{p} \vee \bar{q}} = \bar{\bar{p}} \wedge \bar{\bar{q}} = p \wedge q$
16. $\overline{p \vee \bar{r}} = \bar{p} \wedge \bar{\bar{r}} = \bar{p} \wedge r$
17. $\overline{p \vee q \vee r} = \bar{p} \wedge \bar{q} \wedge \bar{r}$
18. $\overline{\bar{p} \wedge \bar{q} \wedge \bar{r}} = \bar{\bar{p}} \vee \bar{\bar{q}} \vee \bar{\bar{r}} = p \vee q \vee r$
19. $\overline{(\overline{q \vee r}) \wedge \bar{s}} = (\overline{\overline{q \vee r}}) \vee \bar{\bar{s}} = (q \vee r) \vee s = q \vee r \vee s$
20. $\overline{(\overline{p \wedge q}) \vee (\overline{r \wedge s})} = (\overline{\overline{p \wedge q}}) \wedge (\overline{\overline{r \wedge s}}) = p \wedge q \wedge r \wedge s$

LESSON 5: INTRODUCTION TO CIRCUITS

OBJECTIVES:

The student will:

- correctly name a logic gate when its symbol is given.
- provide the output (0 or 1) of a circuit containing combinations of gates, when given the input vector.
- provide the input vectors producing a given output for a circuit containing combinations of gates.

PERIODS RECOMMENDED:

One

OVERVIEW AND REMARKS:

The students might enjoy seeing what a gate looks like during this presentation. You might want to obtain a few from the Science teacher to pass around.

KEY--PROBLEM SET 5:

1. OR
2. NAND
3. INVERT
4. NOR
5. AND
6. a. 0
   b. 0
   c. 0
   d. 1
   e. NOR

7. a.

INPUTS p	q	OUTPUT
1	1	0
1	0	1
0	1	1
0	0	1

   b. NAND

8. a.

INPUTS p	q	OUTPUT
1	1	1
1	0	1
0	1	1
0	0	0

   b. OR

9. a.

INPUTS p	q	OUTPUT
1	1	1
1	0	0
0	1	0
0	0	0

   b. AND

10. [0, 0, 1]
11. [1, 1, 0]
12. [0, 0, 1, 1]
13. [0, 0, 1, 1]
14. [0, 1, 0]
    [0, 1, 1]
    [1, 1, 1]
    [1, 0, 1]
    [0, 0, 1]
15. [1, 0, 1, 0]

LESSON 6: SWITCHING FUNCTIONS

OBJECTIVES:

The student will:

- find the switching functions associated with a given circuit.
- use switching functions to decide when two circuits are equivalent.

PERIODS RECOMMENDED: ONe.

KEY--PROBLEM SET 6:

1. $s \wedge t$
2. $\overline{s \wedge t} = \bar{s} \vee \bar{t}$
3. $\bar{s}$
4. $\overline{s \vee t} = \bar{s} \wedge \bar{t}$
5. $s \vee t$
6. a. $p \wedge q$
   b. $\overline{p \wedge q} = \bar{p} \vee \bar{q}$
   c. NAND
7. a. $\bar{p}$
   b. $\bar{q}$
   c. $\overline{\bar{p} \wedge \bar{q}} = p \vee q$
   d. OR
8. a. $\bar{p}$
   b. $\bar{q}$
   c. $\bar{p} \vee \bar{q}$
   d. $\overline{\bar{p} \vee \bar{q}} = p \wedge q$
   e. AND
9. $\bar{p}$
10. $\bar{p} \vee q$
11. $\bar{q}$
12. $p \wedge \bar{q}$
13. $\overline{p \wedge \bar{q}} = \bar{p} \vee q$
14. yes
15. $\overline{p \wedge q \wedge r \wedge s}$
16. $p \wedge q \wedge r \wedge s$
17. $\bar{p}$
18. $\bar{q}$
19. $\overline{\bar{p} \wedge \bar{q} \wedge r \wedge s}$
20. $\bar{p} \wedge \bar{q} \wedge r \wedge s$

LESSON 7: TRUTH TABLES TO SWITCHING FUNCTIONS

OBJECTIVE:

When given a truth table, the student will find the corresponding switching function.

PERIODS RECOMMENDED:

One

KEY--PROBLEM SET 7:

1. 0, 1
2. $\bar{p} \wedge q$
3. $\bar{p} \wedge \bar{q}$
4. 1, 0, 1
5. $p \wedge \bar{q} \wedge r$
6. $(\bar{p} \wedge q \wedge r) \vee (p \wedge q \wedge \bar{r})$
7. $(p \wedge \bar{q} \wedge r) \vee (\bar{p} \wedge q \wedge \bar{r})$
8. $p \wedge \bar{q} \wedge \bar{r} \wedge s$
9. 0, 1
10. $\overline{\bar{p} \wedge q} = p \vee \bar{q}$
11. $\overline{\bar{p} \wedge \bar{q}} = p \vee q$
12. 0, 0, 1
13. $\overline{\bar{p} \wedge \bar{q} \wedge r} = p \vee q \vee \bar{r}$
14. $\overline{(p \wedge \bar{q} \wedge r) \vee (p \wedge \bar{q} \wedge \bar{r})} = (\bar{p} \vee q \vee \bar{r}) \wedge (\bar{p} \vee q \vee r)$
15. $(p \wedge q) \vee (\bar{p} \wedge \bar{q})$
16. $\overline{(p \wedge \bar{q}) \vee (\bar{p} \wedge q)} = (\bar{p} \vee q) \wedge (p \vee \bar{q})$
17. $(p \wedge \bar{q}) \vee (\bar{p} \wedge q) \vee (\bar{p} \wedge \bar{q})$
18. $\overline{p \wedge q} = \bar{p} \vee \bar{q}$

LESSON 8: SWITCHING FUNCTIONS TO CIRCUITS

OBJECTIVE:

When given a switching function, the student will design a corresponding circuit.

PERIODS RECOMMENDED:

One

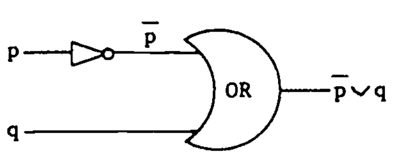
PREPARATION FOR FUTURE LESSON:

You should have a look at Lesson 10 at this point. Some preparations are necessary for that lesson.

KEY--PROBLEM SET 8:

1. AND
2. AND
3. OR
4. AND
5. NAND
6. OR
7. INVERT
8. OR
9. INVERT
10. AND
11. NOR
12. INVERT
13. NOR

14.



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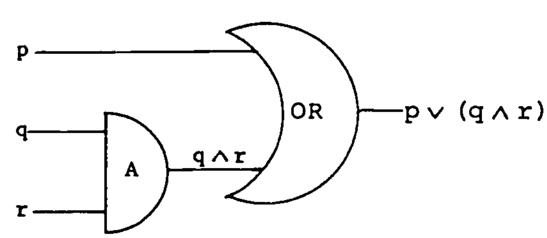
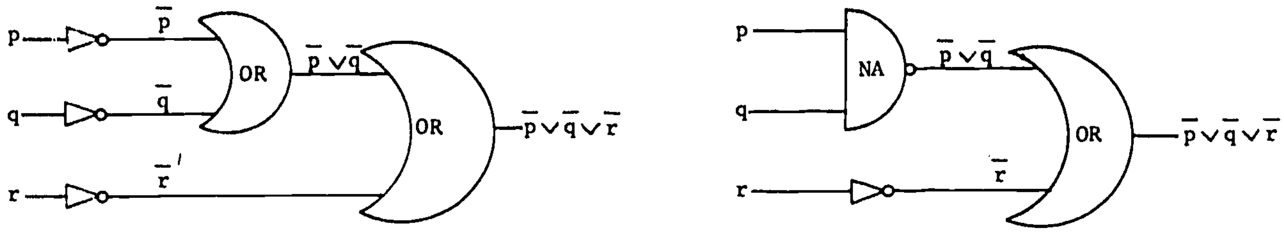


16. There are several possibilities. Two are as follows:



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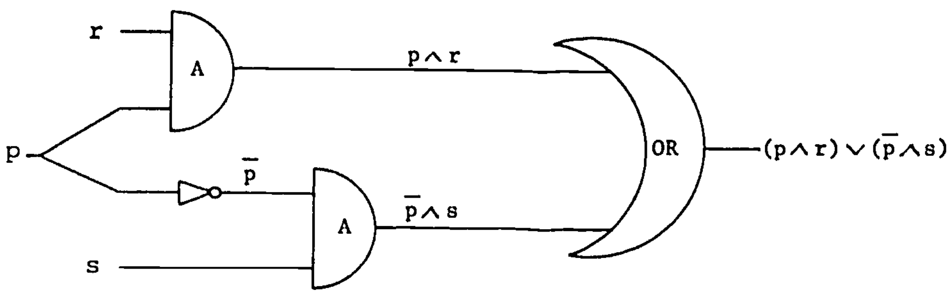
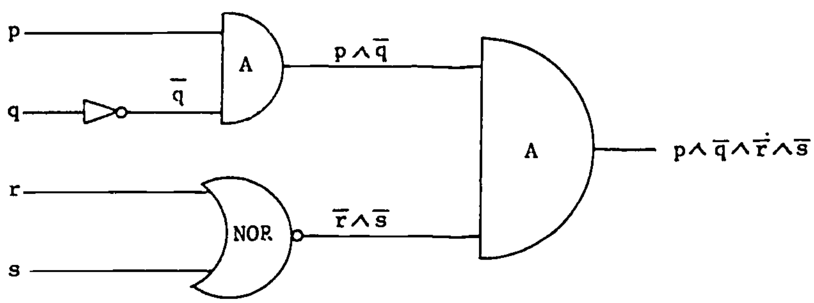


18. One solution is the following.



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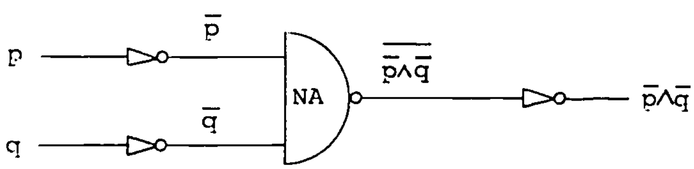

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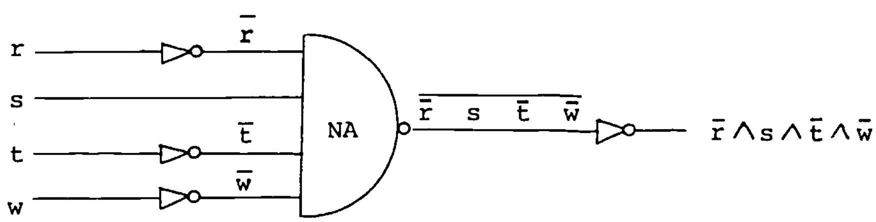

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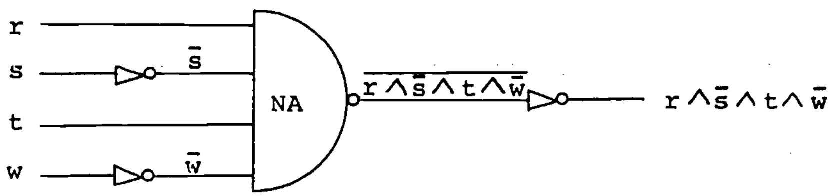
21.



LESSON 9: REVIEW

OBJECTIVE:

The student will solve problems related to the objectives for Lessons 1-8.

PERIODS RECOMMENDED:

One or two.

KEY--REVIEW PROBLEM SET 9:

1. NS
2. S, O
3. S, 1 (don't forget man-made satellites)
4. $\bar{r}$
5. $p\wedge\bar{q}$
6. $\bar{\bar{q}}\wedge\bar{p} = q\wedge\bar{p}$
7. $q\vee p$
8. no

9.

p	q	$\bar{q}$	$p\wedge\bar{q}$	$\overline{p\wedge\bar{q}}$
1	1	0	0	1
1	0	1	1	0
0	1	0	0	1
0	0	1	0	1

10.

p	q	$\bar{p}$	$\bar{p}\vee q$
1	1	0	1
1	0	0	0
0	1	1	1
0	0	1	1

11. a. yes b. $\overline{p\wedge\bar{q}} = \bar{p}\vee\bar{\bar{q}} = \bar{p}\vee q$

12. $\overline{\bar{p}\vee\bar{q}} = \bar{\bar{p}}\wedge\bar{\bar{q}} = p\wedge q$

13. $\overline{p\wedge\bar{q}} = \bar{p}\vee\bar{\bar{q}} = \bar{p}\vee q$

14. $\overline{p\wedge\bar{q}\wedge r} = \bar{p}\vee\bar{\bar{q}}\vee\bar{r} = \bar{p}\vee q\vee\bar{r}$

15. $\overline{(\bar{p}\vee q)\wedge(\overline{\bar{p}\wedge q})} = (\overline{\bar{p}\vee q})\vee(\overline{\overline{\bar{p}\wedge q}}) = (\bar{\bar{p}}\wedge\bar{q})\vee(\bar{p}\wedge q) = (p\wedge\bar{q})\vee(\bar{p}\wedge q)$

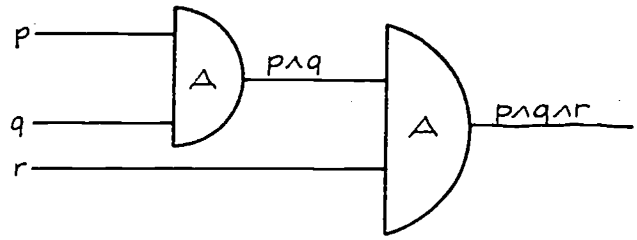
16. a. INVERT, OR b. $3 = \bar{p}$, $4 = \bar{p}\vee q$ c.

p	q	$\bar{p}$	$\bar{p}\vee q$
1	1	0	1
1	0	0	0
0	1	1	1
0	0	1	1

d. 0

17. a. 0 b. none

18. a. [1, 1, 1]   b.



19. a. $\bar{q}$ b. $p \wedge \bar{q}$ c. $(p \wedge \bar{q}) \vee r$

20. 0

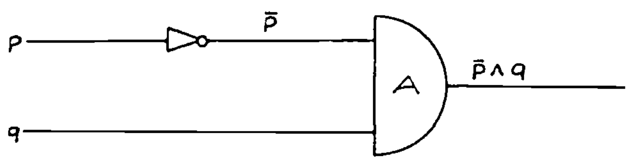
21. a. $\bar{p}$ b. $\bar{p} \vee q$ c. $\bar{r}$ d. $\overline{(\bar{p} \vee q) \wedge \bar{r}}$

22. yes, $\overline{(\bar{p} \vee q) \wedge \bar{r}} = \overline{(\bar{p} \vee q)} \vee \bar{\bar{r}} = (\bar{\bar{p}} \wedge \bar{q}) \vee r = (p \wedge \bar{q}) \vee r$

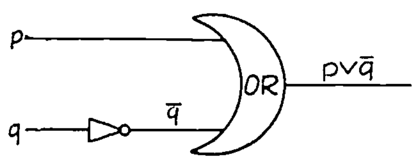
23. $\bar{p} \wedge q$

24. $\overline{\bar{p} \wedge q} = p \vee \bar{q}$

25. a.



b.



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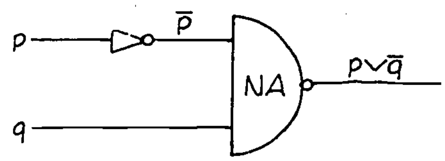


c. depends on circuit diagram

d. depends on circuit diagram

e. See a and b above.

26. $(p \wedge q \wedge r) \vee (\bar{p} \wedge \bar{q} \wedge \bar{r})$

27. $\overline{(p \wedge q \wedge r) \vee (\bar{p} \wedge \bar{q} \wedge \bar{r})} = \overline{(p \wedge q \wedge r)} \wedge \overline{(\bar{p} \wedge \bar{q} \wedge \bar{r})} = (\bar{p} \vee \bar{q} \vee \bar{r}) \wedge (\bar{\bar{p}} \vee \bar{\bar{q}} \vee \bar{\bar{r}}) =$

$(\bar{p} \vee \bar{q} \vee \bar{r}) \wedge (p \vee q \vee r)$

28. INVERT

29. AND

30. OR

31. INVERT

32. NAND

33. INVERT

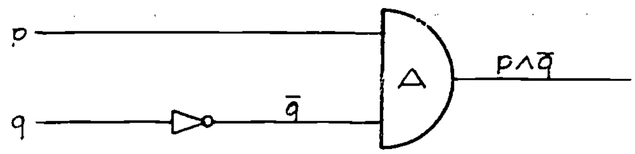
34. OR

14

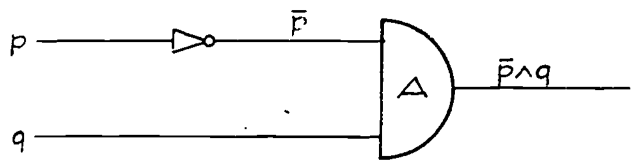
35. NAND

36. NOR

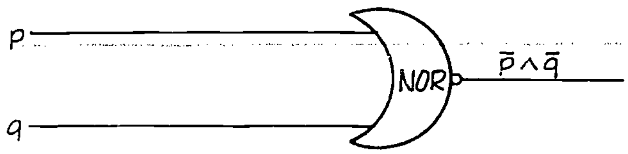
37.



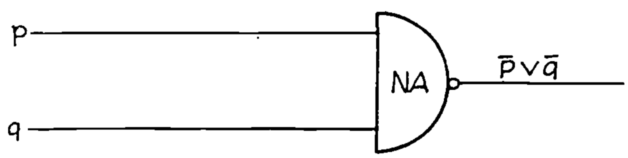
38.



39.



40.



41. a. depends on circuit diagram

    b. See answer to Problem 39.

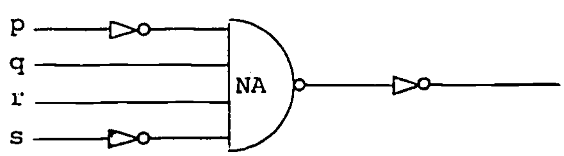
42. a. depends on circuit diagram

    b. See answer to Problem 40.

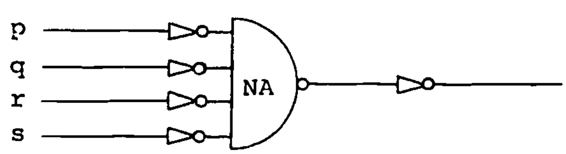
43. [1, 1, 0, 0]

44. [0, 0, 0, 1, 1]

45.



46.



LESSON 10: ANGLES AND ANGLE MEASURE

OBJECTIVES:

The student will:

- recognize angles and use a protractor to measure them.
- interconvert degree, radian, cycle and revolution measures.

PERIODS RECOMMENDED: One.

SUPPLIES:

Transparency Master IV-M-10 (at end of book)

Protractor for each student, preferably the type with radian, as well as degree measure.

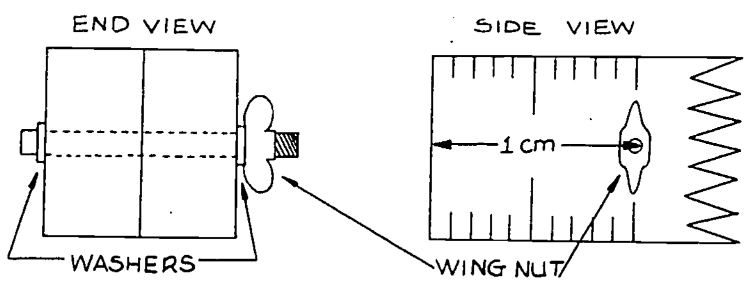
Meter stick goniometers--one for 3 students (construction details described below).

OVERVIEW AND REMARKS:

You might be curious about the inclusion of the cycle measure of angles. Later in the study of sine waves and sound it will be very useful to have the idea of a cycle well mastered, cycles per second being a fundamental measure of frequency.

The transparency master may be used to prepare a transparency for overhead projection.

Following Problem Set 10, Activity 10 provides the students with an opportunity to measure one another's range of motion. The activity requires about ten "goniometers." All that needs to be done to construct the Biomedical goniometer is to drill holes near the ends of a pair of meter sticks, yardsticks, laths or other flat wooden sticks that are about one yard long (no linear measurements are required) and bolt them together with the appropriate sized bolt. We used a $\frac{1}{8}$" diameter x $\frac{3}{4}$" long bolt and drilled a $\frac{1}{8}$" diameter hole about 1 cm from the end. A pair of washers and a wing nut complete the hardware. The arrangement is diagrammed below.



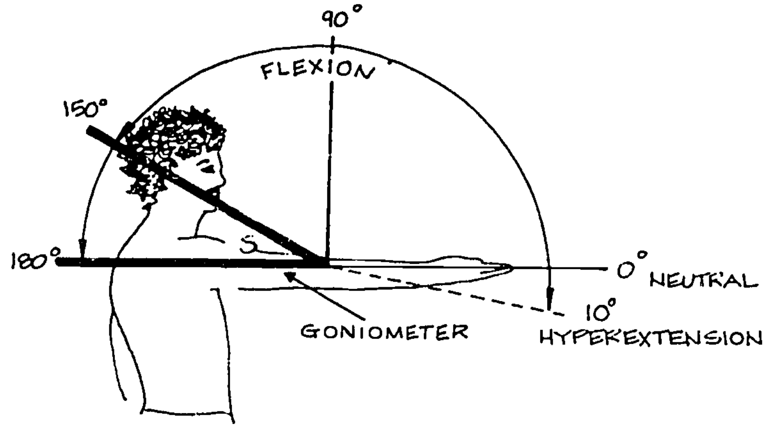
When using the goniometer, it is often easier to measure the supplement of the indicated range of motion than to measure the angle directly. Consider the situation described in the illustration on the following page.

Once ∠S is measured, it may be subtracted from 180° to find the range of motion.

Students may most conveniently work in groups of three, one to be the subject, one to hold the goniometer in position and one to tighten the wing nut once the instrument is in position and to measure the angle.

We suggest that you demonstrate a couple of goniometer measurements with the assistance of two students before the class starts Activity 10.

FLEXION and HYPEREXTENSION



Position of Goniometer to Measure ∠S.

PREPARATION FOR FUTURE LESSONS:

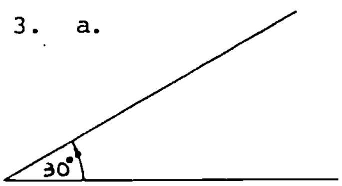
A suggested activity in Lesson 11 calls for two slide projectors and two identical slides. If you are interested in pursuing the activity, you will need to begin collecting the equipment and slides.

KEY--PROBLEM SET 10:

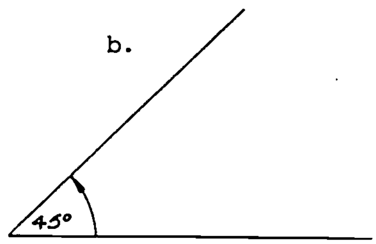
1. a. 40° e. 45°
   b. 121° f. 134°
   c. 62° g. 450°
   d. 180°

2. a. 51° d. 131°
   b. 30° e. 309°
   c. 50° f. Yes

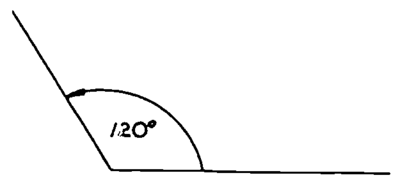
3. a.



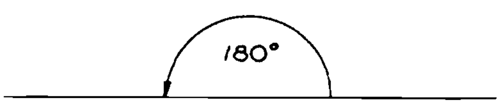
b.



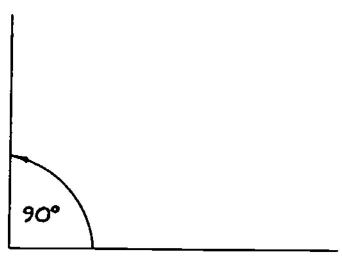
c.



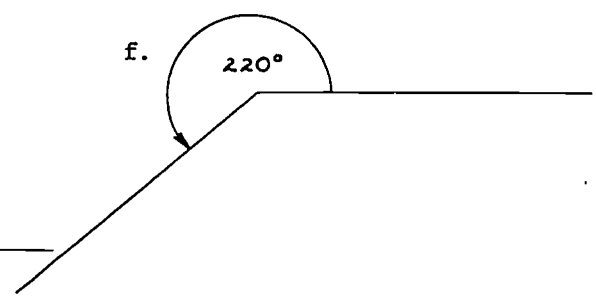
d.



e.



f.



4. a. 2π meters
   b. $\frac{\pi}{2}$ meters
   c. π meters
   d. $\frac{4\pi}{3}$ meters
   e. $\frac{5\pi}{3}$ meters
   f. 45° or $\frac{1}{8}$ rev
   g. 120° or $\frac{1}{3}$ rev
   h. 315° or $\frac{7}{8}$ rev

5. radian

6. a. 360° 7. a. 180° 8. a. $\frac{\pi}{2}$ c. $\frac{\pi}{6}$ e. $\frac{11\pi}{6}$

b. 2π b. 90° b. π d. $\frac{11\pi}{9}$

c. 360°, 2π c. 135°

9. a. 720°, 4π radians c. 900°, 5π radians e. 420°, $\frac{7\pi}{3}$ radians

b. 270°, $\frac{3\pi}{2}$ radians d. 240°, $\frac{4\pi}{3}$ radians f. 450°, $\frac{5\pi}{2}$ radians

10. a. $\frac{2\pi}{3}$ radians b. $\frac{3\pi}{2}$ radians c. 3π radians d. $\frac{5\pi}{2}$ radians

11. 40π radians

12. 120π radians

LESSON 11: SIMILAR FIGURES

OBJECTIVES:

The student will derive and use a set of mathematical tests to determine whether two figures are similar.

SUPPLIES:

2 slide projectors

2 identical slides

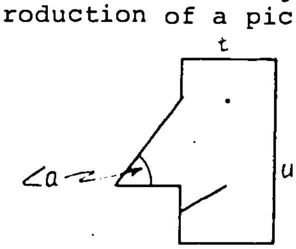
ruler and protractor (one per student)

slide rules or electronic calculators to calculate ratios (optional)

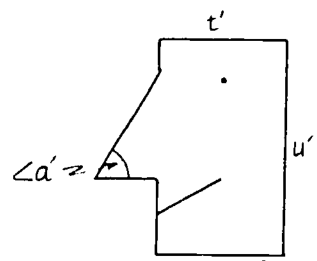
PERIODS RECOMMENDED: One

OVERVIEW AND REMARKS:

The basic thrust of this lesson is to get the class to deduce from their own observations statements roughly equivalent to the following ones about distortion in the reproduction of a picture.



Picture A



Picture A'

1. If $\frac{t'}{u'} \neq \frac{t}{u}$ then Picture A' is a distorted reproduction of Picture A.

2. If $\angle a' \neq \angle a$ then Picture A' is a distorted reproduction of Picture A.

The development of this lesson follows a discovery approach. The <u>recognition</u> of distorted reproductions versus faithful magnifications should come easily to the students. Hence the main difficulty will be to lead the students to verbalize mathematical tests for distorted and true reproductions. The following activities should provide a framework for development of the material. The concepts of this lesson will receive a more rigorous treatment in the following lesson on similar triangles.

TEACHER ACTIVITIES:

A. If practical considerations have allowed you to obtain two projectors and two identical slides, they can be used to advantage in introducing this material. Project the identical slides from both projectors. By leaving one projector stationary and varying both the distance away from the screen and the angle at which the image strikes the screen (or wall), you can produce both distorted and undistorted reproductions of the slide being projected from the stationary projector. As the students recognize the distorted images, ask them how they might specifically describe the nature of the distortions. If the students begin talking about the relative dimensions, they might be invited to use a meter stick to make measurements of the dimensions of the images. The measurements could then be compared, leading to the notion of examining ratios and so on.

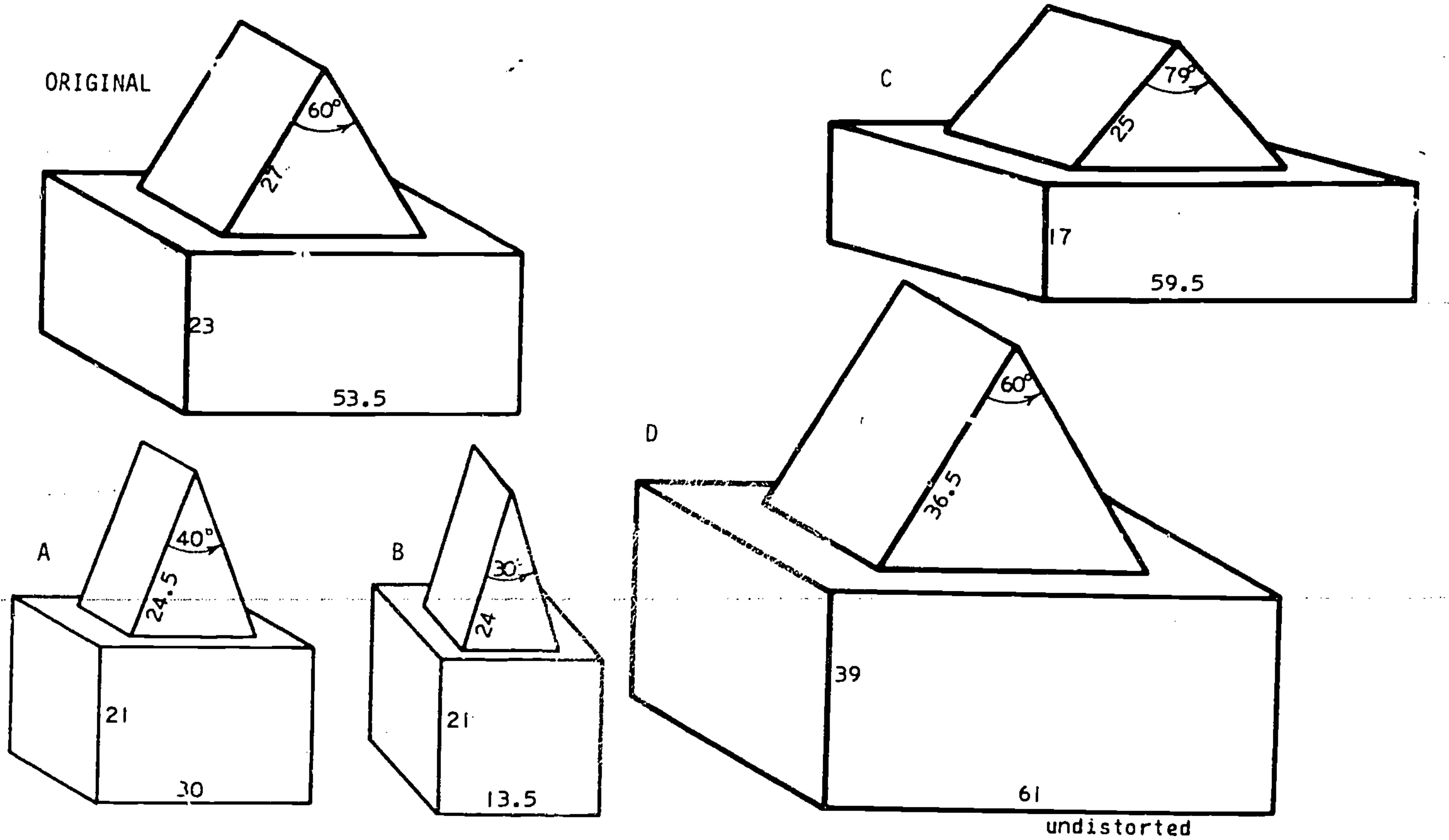
B. As a second step in the discovery process, refer the students to Worksheet 11. Each of the pages has a basic picture called the "original" and several reproductions, some of which are distorted and some not. Begin by polling the students on which figures are distortions.

Devote the remainder of the period to having the students produce mathematical arguments to support their contentions. You will need to choose a proper balance between leadership and discovery in this part of the lesson, depending on the emergence of clearly defined mathematical tests for distortion before this point. At one extreme, the students could be challenged to use ruler and protractor in any plausible way to support their claims. At the other end of the spectrum, they could be instructed to make specific measurements, take their ratios, and then compare results with the "original." In either case, the end result should involve making some identical measurement on all the figures and then discovering that corresponding measurements on the undistorted figures are approximately proportional to those of the original. Note that the key for Worksheet 11 gives some of the major measurements. Linear dimensions are in millimeters and angles in degrees. You will find a slide rule or an electronic calculator invaluable in calculating ratios. In this part of the lesson the following major points should be kept in mind.

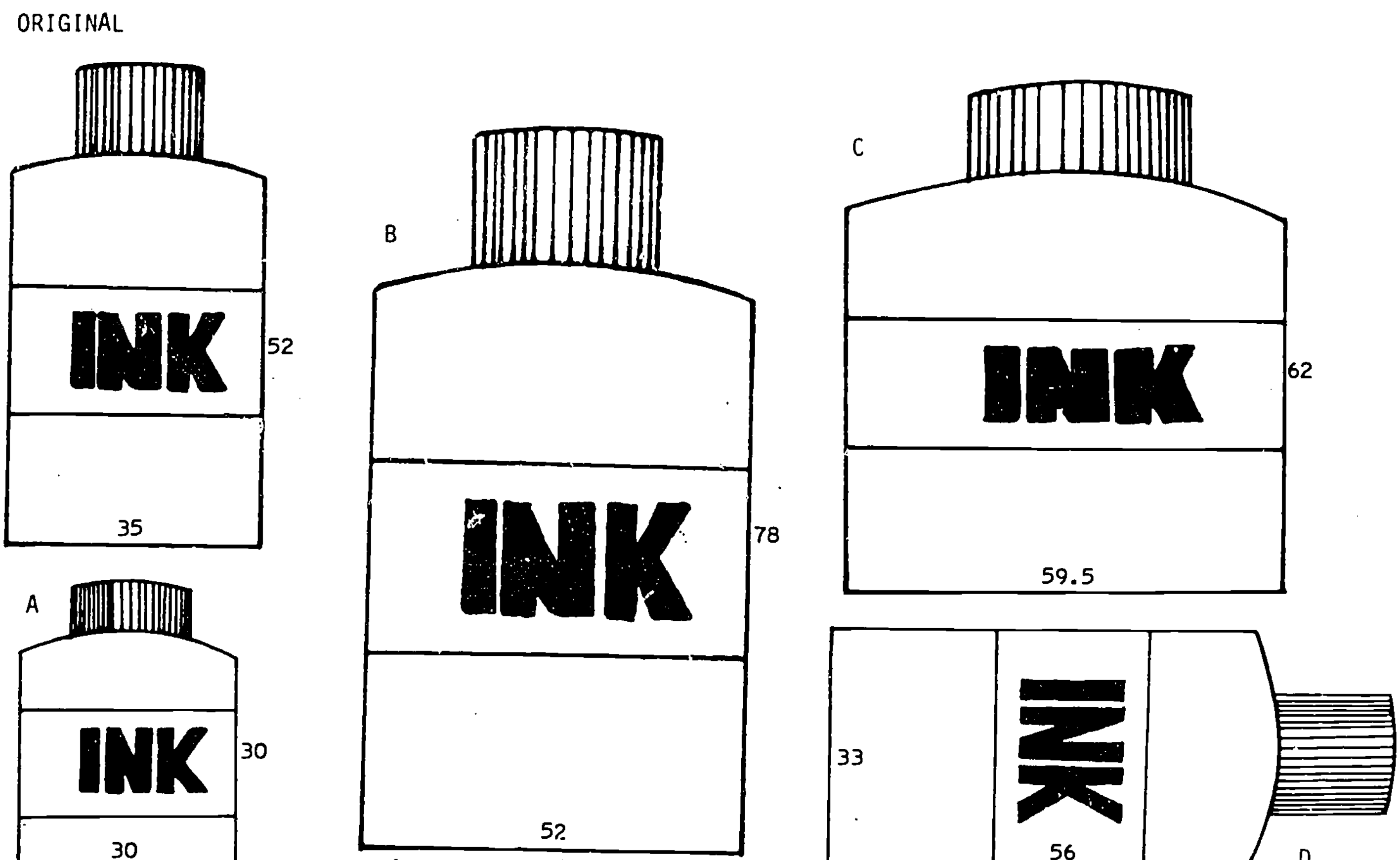
1. There will be the usual errors in measurement. Even though the similar figures have been generated photographically, the ratios of the dimensions given in the key will not be identical between two similar figures, because of measurement error. Also there is the problem of "perceptual threshold" here; the student may be unable to distinguish visually between two figures that are in fact mathematically dissimilar.

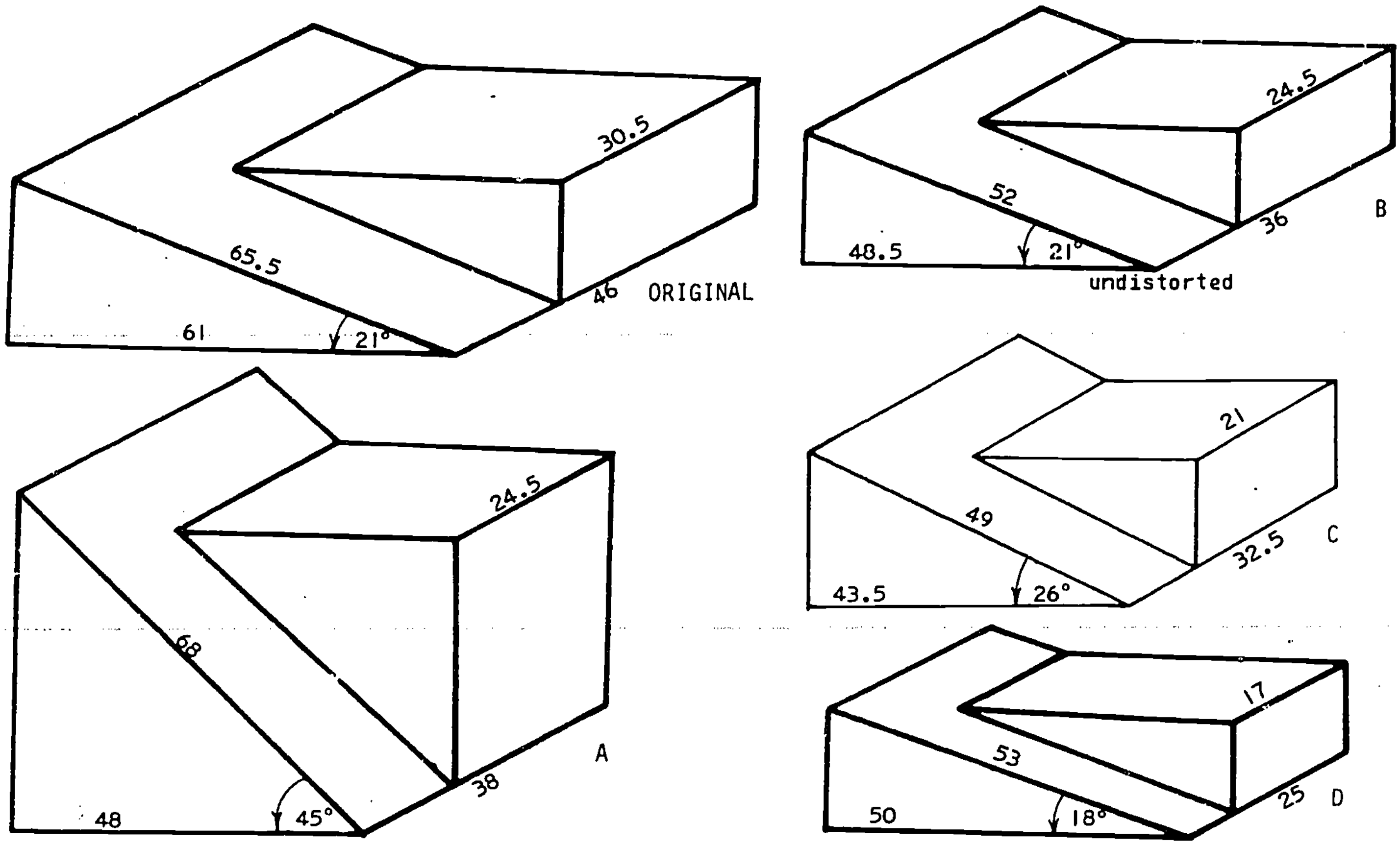
2. There is a lot of computation involved, and a choice must be made either to have the class concentrate on a couple of sets of figures of the Worksheet or to divide the class into study groups.

3. Circulation will be important in this part of the lesson. Ratios can be checked via slide rule or electronic calculator and students who are floundering can be given more direction.

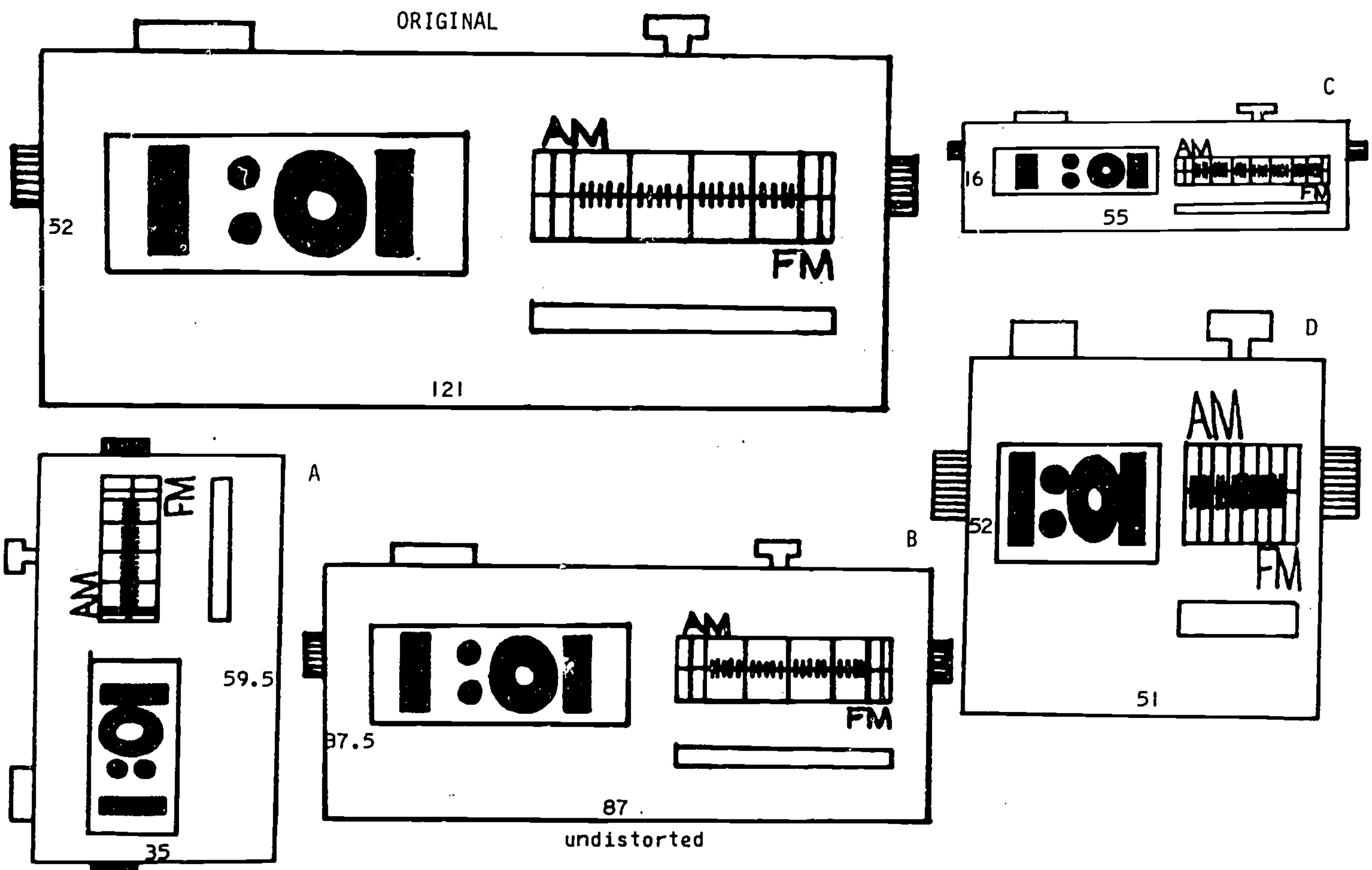


WORKSHEET 11





ORIGINAL
30.5
65.5
46
61
21°
B
24.5
52
36
48.5
21°
undistorted
A
24.5
68
38
48
45°
C
21
49
32.5
43.5
26°
D
17
53
25
50
18°



ORIGINAL
AM
FM
52
121
C
AM
FM
16
55
D
AM
FM
52
51
A
FM
AM
59.5
35
B
AM
FM
87.5
87
undistorted

15b

LESSON 12: SIMILAR TRIANGLES

OBJECTIVES:

The student will:

- identify theorems that establish similarity for given pairs of triangles.
- use the properties of similarity to determine unknown parts of triangles.

PERIODS RECOMMENDED: One.

SUPPLIES:

Transparency Masters IV-M-12a,b

OVERVIEW AND REMARKS:

Problems 14 and 15 suggest real activities that could be set up. You may wish to have your students actually determine the height of objects such as flagpoles, buildings and trees by similar methods.

Note that Transparency IV-M-12b includes answers. They should be concealed from the students until the appropriate time.

KEY--PROBLEM SET 12:

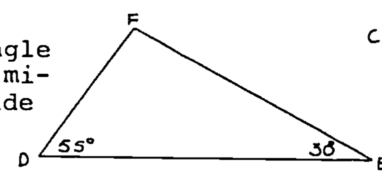
1. Two similar triangles have equal corresponding angles and proportional corresponding sides.

2. If two angles of one triangle are equal to two corresponding angles of another triangle, the triangles are similar.

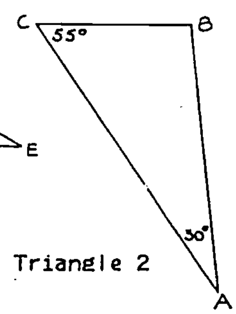
3. If two pairs of corresponding sides are proportional, and the included angles are equal, then the two triangles are similar.

4. If three pairs of corresponding sides are proportional, then the triangles are similar.

5. In the diagram at right, side AB in Triangle #2 corresponds to side EF in Triangle #1. Similarly, side BC corresponds to side FD, and side AC corresponds to side ED.



Triangle 1



Triangle 2

6. If two angles of one triangle are equal to the corresponding angles of another triangle, the triangles are similar. (AA)

7. If two sides of one triangle are proportional to two sides of another triangle, and if the angles between those sides are equal, the triangles are similar. (SAS)

8. If the three sides of one triangle are proportional to the corresponding three sides of another triangle, the triangles are similar. (SSS)

9. a. 2
   b. 3

10. a. 9
    b. 12

11. a. $s = 6$
    b. $t = 2\frac{1}{2}$

12. a. If 2 angles of one triangle are equal to 2 angles of another, the triangles are similar. (AA)

13. a. CE b. $5\frac{5}{11}$

14. a. Both are 90° angles.

b. same angle

c. If 2 angles of one triangle equal 2 angles of another, the triangles are similar. (AA)

d. proportional

e. 20 meters

15. 18 meters

16. a. Yes

b. If 2 angles of one triangle equal 2 angles of another, the triangles are similar. (AA)

c. DA

d. 6 or BC

e. $7\frac{1}{2}$

17. $9\frac{1}{3}$

18. a. decreases

b. A greater object-to-image distance increases the magnification.

c.

	SI	SO	I_1I_2	O_1O_2
(1)	40	32	15	12
(2)	80	72	$13\frac{1}{3}$	12
(3)	40	24	20	12
(4)	144	128	$13\frac{1}{2}$	12

d. 8 cm

e. 16 cm

LESSON 13: THE PYTHAGOREAN THEOREM AND ITS CONVERSE

OBJECTIVES:

The student will:

- use the relationship $a^2 + b^2 = c^2$ to determine the value of the third variable when any two are known.
- decide whether a given triangle is a right triangle when given the lengths of the three sides.
- rationalize denominators.

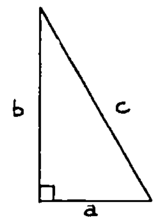
PERIODS RECOMMENDED: One or two.

OVERVIEW AND REMARKS:

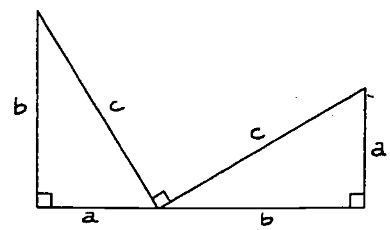
A proof of the Pythagorean Theorem does not appear in the student material. However, if you want to present one, the following one may interest you.

This historical tidbit was adapted from J. Eves, In Mathematical Circles, #332, Prindhe, Weber, & Schmidt, Boston, 1969. President James Garfield (1831-1881) is credited with the discovery of a new and elegant proof of the Pythagorean Theorem. According to the story, he was discussing geometry with some other members of the House of Representatives before he became president and hit upon the idea. The proof was subsequently published in the New England Journal of Education.

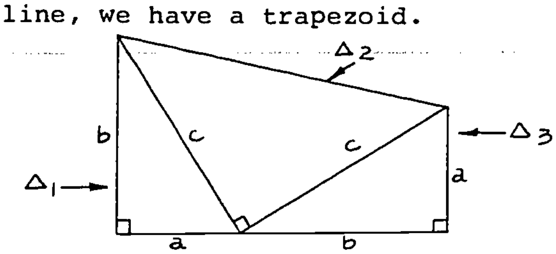
We start with the triangle below; we want to show that $a^2 + b^2 = c^2$.



We add another right triangle congruent to the first.



Adding one more line, we have a trapezoid.



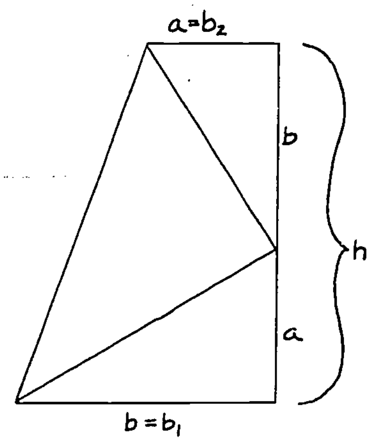
We compute the area of the trapezoid in two different ways.

1. Use the formula for the area of trapezoids developed in Section 3 of Unit I.

$$\text{Area of trapezoid} = \frac{(b_1 + b_2)}{2} \cdot h$$

In this case, $b_1 = b$, $b_2 = a$, $h = b + a$. Therefore,

$$\text{Area of trapezoid} = \left(\frac{b+a}{2}\right) \cdot (b+a)$$



2. Use the fact that the area of the trapezoid will also be given by the equation

$$\text{Area of trapezoid} = (\text{A of } \Delta_1) + (\text{A of } \Delta_2) + (\text{A of } \Delta_3)$$

where

$$\text{A of } \Delta_1 = \frac{ba}{2}$$

$$\text{A of } \Delta_2 = \frac{c^2}{2}$$

$$\text{A of } \Delta_3 = \frac{ba}{2}$$

Therefore,

$$\text{Area of trapezoid} = \frac{ba}{2} + \frac{c^2}{2} + \frac{ba}{2}$$

The right-hand parts of the two equations must be equal because they are both expressions for the area of the trapezoid; therefore,

$$(b + a)^2 = ba + c^2 + ba$$

$$b^2 + 2ab + a^2 = 2ba + c^2$$

$$b^2 + a^2 = c^2$$

Since a trapezoid can be constructed for any given values of a and b, the theorem of Pythagoras is proven.

KEY--PROBLEM SET 13:

1. 5
2. 13
3. 8
4. 24
5. 1
6. $\frac{\sqrt{5}}{3}$
7. $\frac{\sqrt{6}}{6}$
8. Yes, because $1^2 + 1^2 = 2$
9. No, because $5^2 + 6^2 = 25 + 36 = 61$ and $61 \neq 8^2$
10. Yes, because $5 + 17 = 22$
11. a. Only when a and/or b = 0.

    b. No, because $\sqrt{7^2 + 24^2} = \sqrt{49 + 576} = \sqrt{625} = 25$

    $\sqrt{7^2} + \sqrt{24^2} = 7 + 24 = 31$

    $25 \neq 31$
12. 50 meters
13. The diagonal $= \sqrt{76^2 + 41^2} \approx 86.35$ cm

    Since the cane (87 cm) is longer than the diagonal (86.35 cm), the cane won't fit on the bottom of the trunk.
14. 100 km
15. ~11.66 m
16. ~91.42 cm
17. x = 60 m

LESSON 14: REVIEW

OBJECTIVE:

The student will solve problems related to the objectives for Lessons 10 through 13.

PERIODS RECOMMENDED: One or two.

PREPARATION FOR FUTURE LESSONS:

Problems 9 through 11 in Problem Set 15 describe how to play a polar form of Tic-Tac-Toe. You may wish to reproduce some playing grids for use in Lesson 15.

KEY--PROBLEM SET 14:

1. a,b
2. a. T b. F c. F d. F e. T f. F g. T h. T
3. a. π b. 2π c. $\frac{\pi}{3}$ d. $\frac{\pi}{12}$ e. $\frac{7\pi}{3}$
4. a. 20° b. 1° c. 270° d. 1440° e. 180° f. 288° g. 828°
5. a. 5π radians b. 900° c. 2.5 rev d. 2.5 cycles
6. a. AA b. SSS c. ns--the sides are not in the same proportion. d. SAS c. ns--corresponding angles are not equal.

7. 24

8. $9\frac{1}{2}$

9. 15

10. 82 cm

11. a. $2\sqrt{2}$
 b. 11
 c. $10\sqrt{7}$
 d. $3\sqrt{11}$
 e. $3a^2\sqrt{3}$
 f. $10a^3\sqrt{a}$
 g. 10^4

12. a. $\frac{\sqrt{11}}{11}$
 b. $\frac{2\sqrt{7}}{7}$
 c. $\frac{5\sqrt{2}}{2}$
 d. $\frac{\sqrt{3}}{3}$

13. a. 10
 b. 3
 c. 5

14. a. No. $7^2 + 7^2 = 98$ and $10^2 = 100$
 b. Yes. $\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{8}}\right)^2 = \frac{1}{2} + \frac{1}{8} = \frac{5}{8}$
 and $\left(\frac{\sqrt{10}}{4}\right)^2 = \frac{10}{16} = \frac{5}{8}$

LESSON 15: POLAR COORDINATES AND POLAR VECTORS

<u>OBJECTIVES</u>:

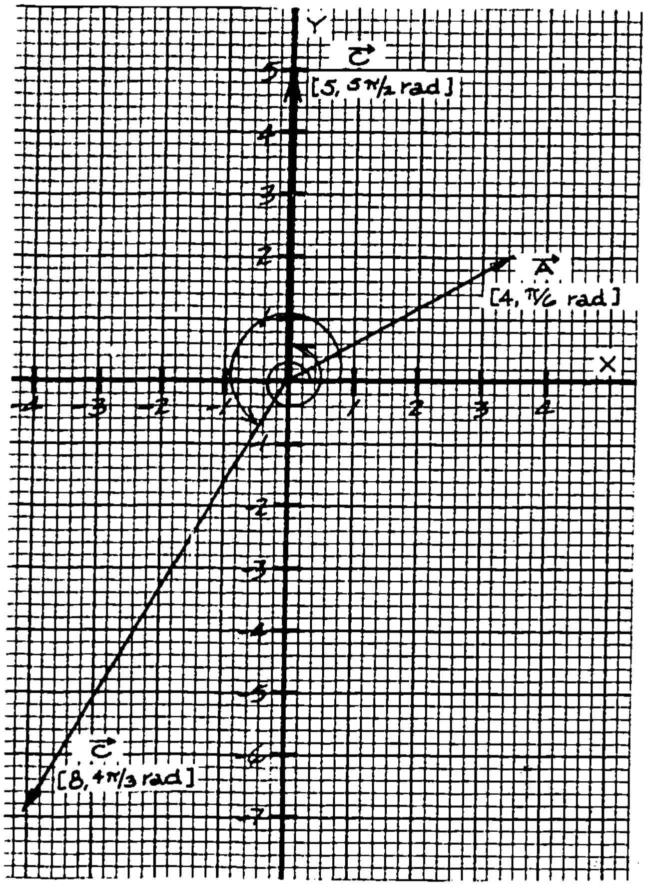
The student will:

- draw vectors when given their polar components.
- graph points when given their polar coordinates.
- use graphical methods to interconvert polar and rectangular representations of points and vectors.

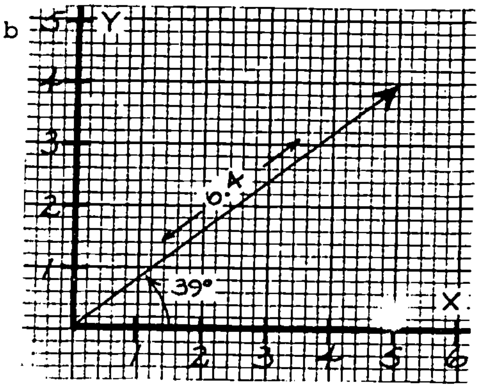
<u>PERIODS RECOMMENDED</u>: One.

KEY--PROBLEM SET 15:

1.



2. a and b

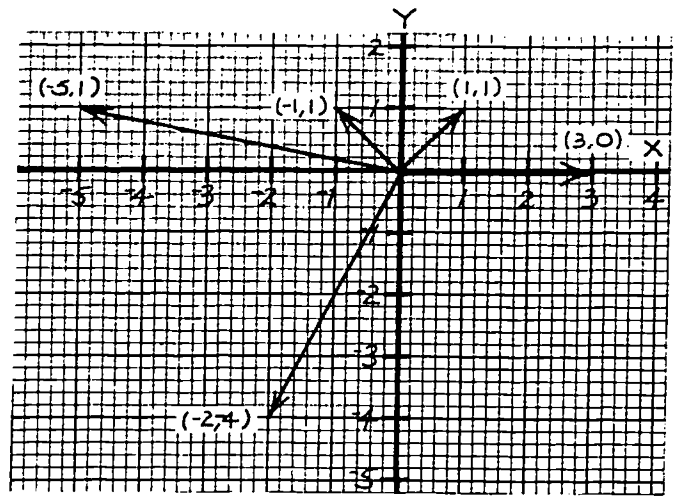


c. length ≈ 6.4 cm

d. angle ≈ 39°

e. [6.4, 39°]

3.



a. ($\sqrt{2}$, 45°)

b. ($\sqrt{2}$, 135°)

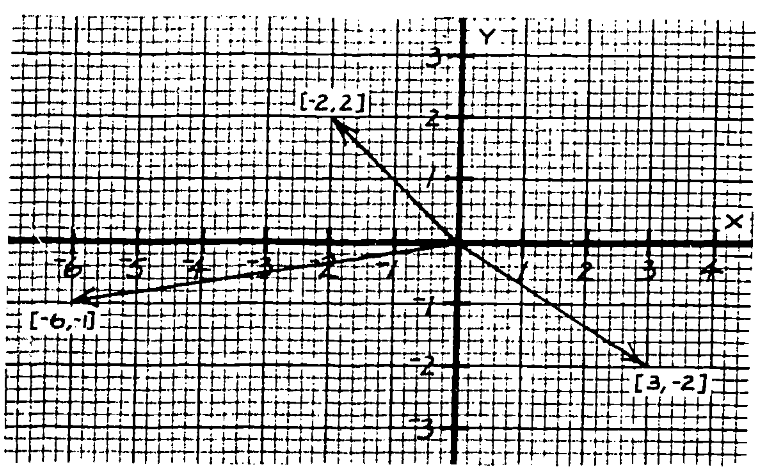
c. ($\sqrt{26}$, 169°)

d. (3, 0°)

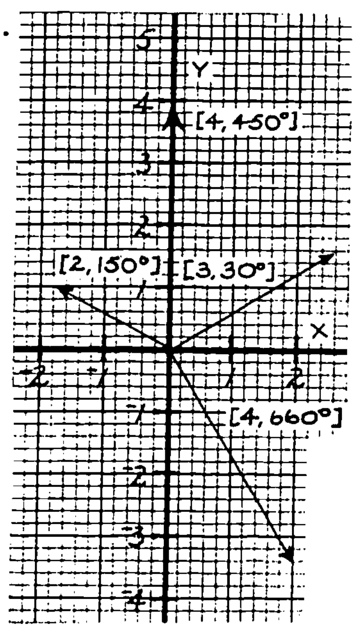
e. ($2\sqrt{5}$, 244°)

4. a. (See graph below.)

b. (1) [$2\sqrt{2}$, 135°] (2) [$\sqrt{37}$, 189°]
(3) [$\sqrt{13}$, 326°]



5.



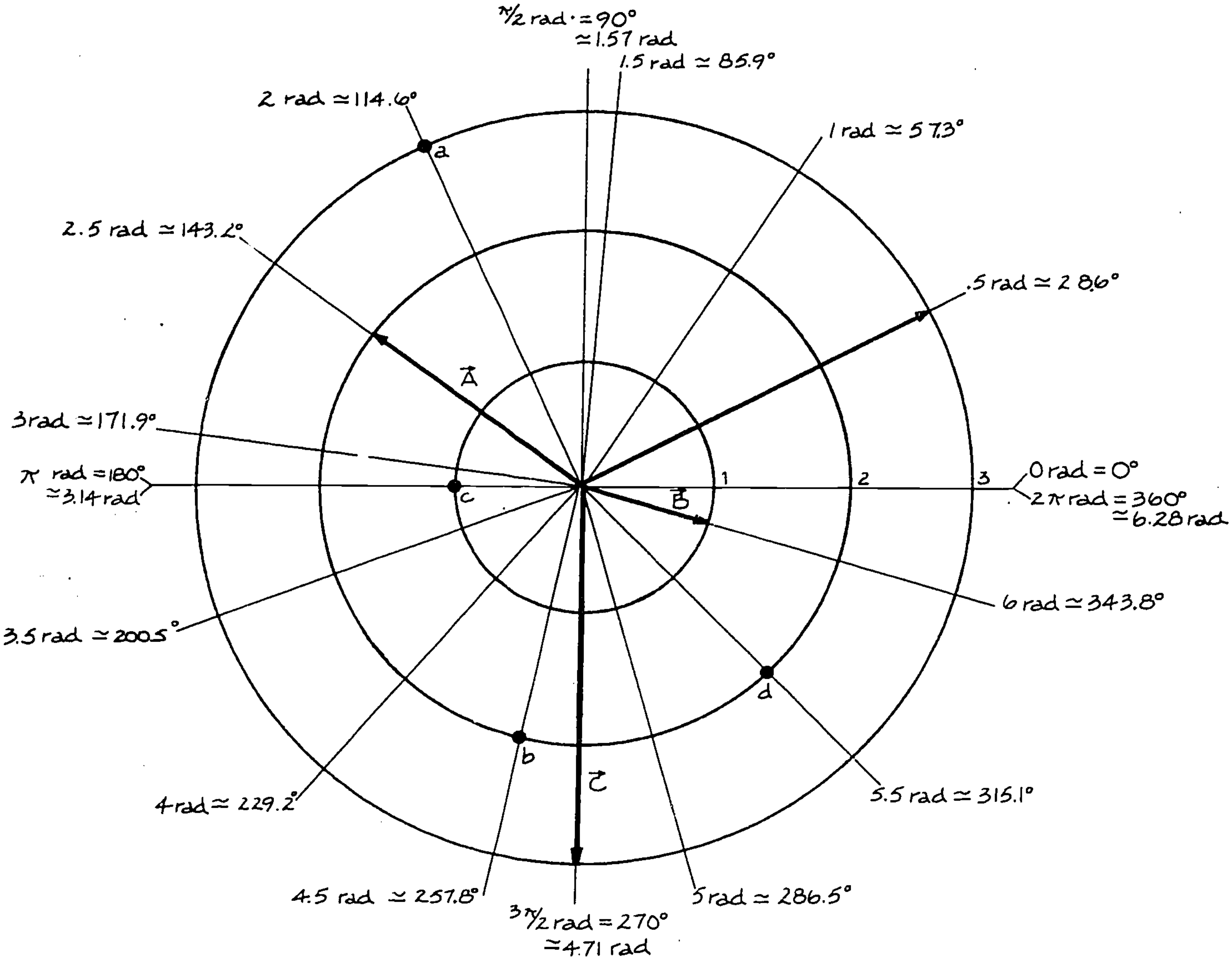
a. (2.6, 1.5)

b. (0, 4)

c. (-1.7, 1)

d. (2, -3.5)

6, 7, and 8



9. a. $(2, \frac{\pi}{2})$, $(2, \frac{2\pi}{3})$, $(2, \frac{5\pi}{6})$

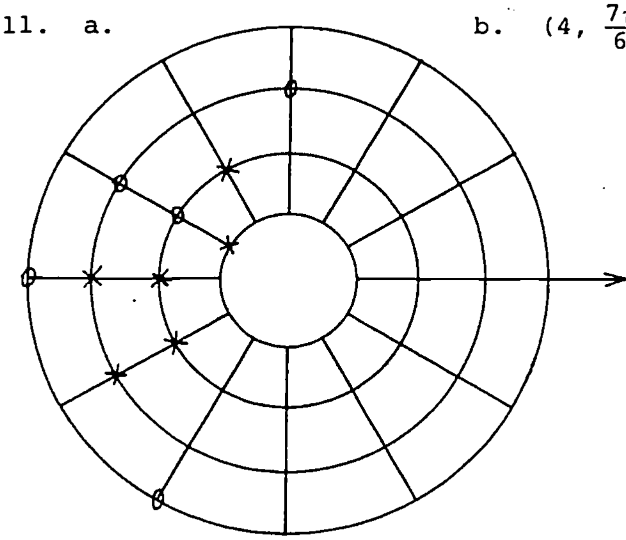
   b. $(1, \frac{\pi}{6})$, $(2, \frac{\pi}{3})$, $(3, \frac{\pi}{2})$

   c. $(2, \pi)$

   d. $(4, \frac{2\pi}{3})$

10. $(1, \frac{\pi}{6})$

11. a.



b. $(4, \frac{7\pi}{6})$

LESSON 16: POLAR VECTORS AND FORCES

OBJECTIVE:

The student will use graphical methods to add vectors stated in polar terms.

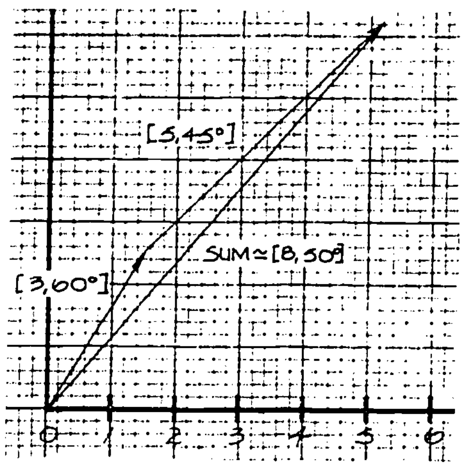
PERIODS RECOMMENDED: One or two.

OVERVIEW AND REMARKS:

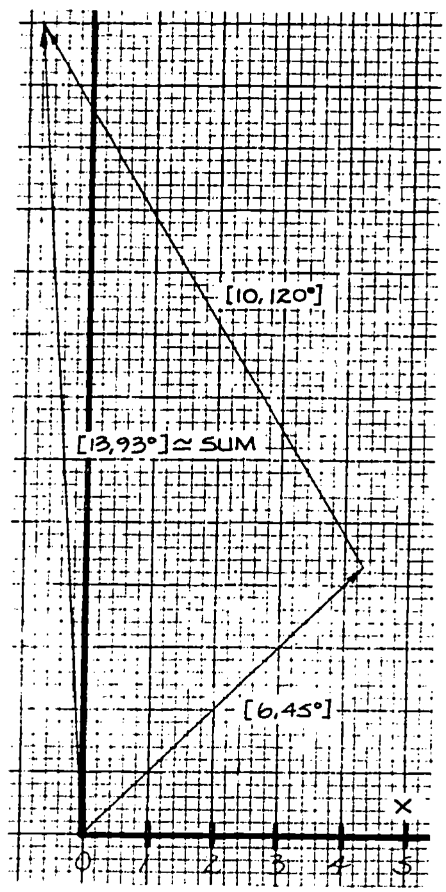
This is important material for the science course. In Science Unit VI it will be necessary to represent forces acting on the body during collisions in terms of vectors. The material here begins to lay the groundwork for this later material.

KEY--PROBLEM SET 16:

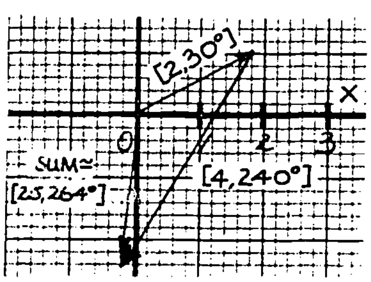
1.



2.

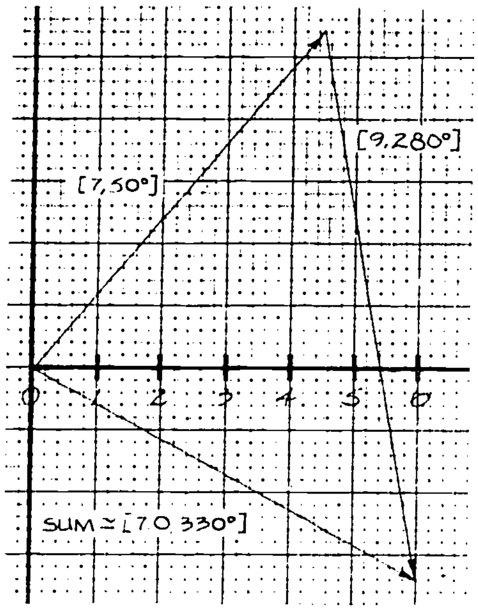


3.

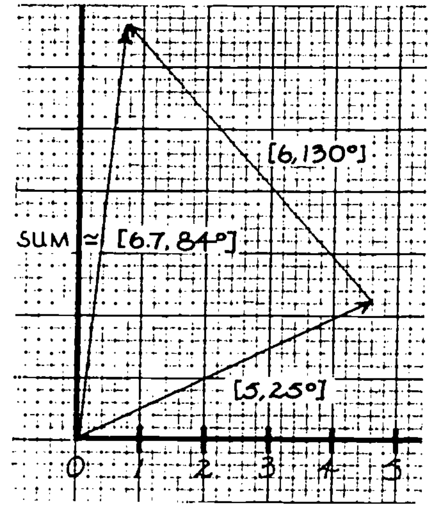


29

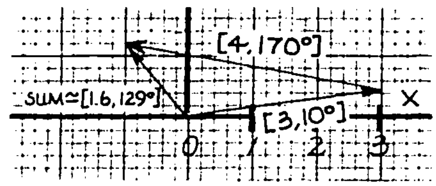
4.


[9,280°]
[7,50°]
SUM ≃ [7.0, 330°]
0
1
2
3
4
5
6

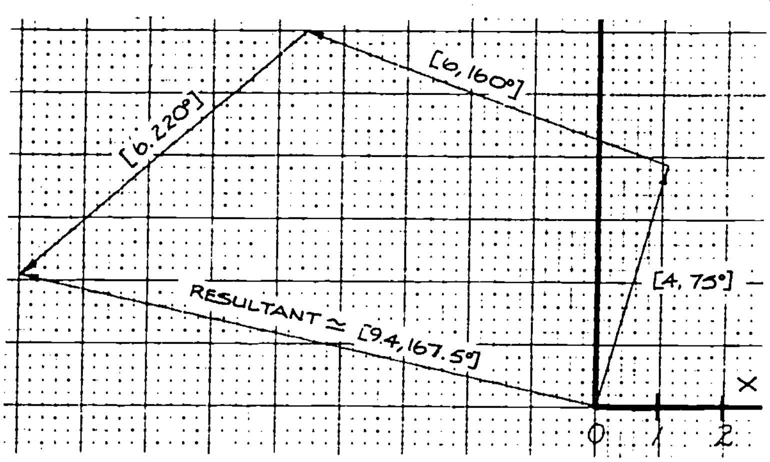
5.


[6,130°]
SUM ≃ [6.7, 84°]
[5,25°]
0
1
2
3
4
5

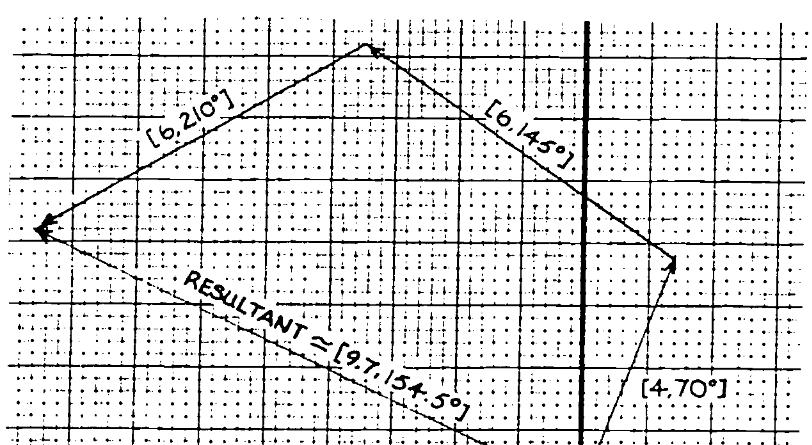
6.


[4,170°]
SUM ≃ [1.6, 129°]
[3,10°]
X
0
1
2
3

7.


[6,160°]
[6,220°]
[4, 75°]
RESULTANT ≃ [9.4, 167.5°]
X
0
1
2

8.


[6,210°]
[6,145°]
RESULTANT ≃ [9.7, 154.5°]
[4,70°]
X
0
1
2
3

LESSON 17: COSINE AND SINE

OBJECTIVE:

The student will:

- obtain the cosine and sine of specific angles by graphical methods.
- describe the behavior of the signs of these two functions in the four quadrants.

PERIODS RECOMMENDED: Two.

SUPPLIES:

Transparency Masters IV-M-17a,b

OVERVIEW AND REMARKS:

Below is described a classroom activity that you may wish to lead to generate a brief table of trigonometric functions. Transparency Master IV-M-17a is a blank table. The project of the day is to enter the appropriate values in the table.

Display Transparency IV-M-17b, and instruct the students to reproduce it on their own graph paper. The radius of the circle should be 10 cm, which means that the circle will have to extend one cm beyond the grid on each side. The circles will be used in making measurements, so they should be constructed as carefully as possible.

Have each student use a protractor to measure angles of 0°, 30°, 45° and 90°, and then draw the polar vectors [1, 0°], [1, 30°], [1, 45°] and [1, 90°]. The vectors should originate at the origin and terminate on the unit circle. You can use a grease pencil on Transparency IV-M-17b to demonstrate these constructions. The result should look like Figure 1 on the following page.

Explain that in each case the x- and y-coordinates of the point where the vector meets the circle will be defined respectively as the cos and sin of the given angle. Make the point that we can do this for any angle.

If you want to find cos θ and sin θ you first draw the polar vector [1, θ]. The x-coordinate of the point where the vector meets the unit circle is defined to be cos θ and the y-coordinate is defined to be sin θ.

Now have the students read from their graphs the x- and y-coordinates for each angle and enter the results with a grease pencil in the BRIEF TABLE OF TRIG FUNCTIONS (Transparency IV-M-17b). The values should be expressed to two decimal places. This should present no problems, since each small division on the graph paper represents .02.

Direct the students to rotate their graphs 90° clockwise and mark off angles of 60° and 90° in Quadrant II. Figure 2 (next page) shows what should result. Ask the students what polar vectors they have in fact constructed--[1, 150°] and [1, 180°].

Ask the class again to read off the x- and y-coordinates of the points so that they can be entered in the BRIEF TABLE OF TRIG FUNCTIONS. Remind them that the "x" values or cosines will be negative since they are in Quadrant II.

Continue the process outlined above until the BRIEF TABLE OF TRIG FUNCTIONS is completed.

To save time, you may wish to divide the class into groups and assign some angles to each group. Then record the results of the different groups in the BRIEF TABLE OF TRIG FUNCTIONS.

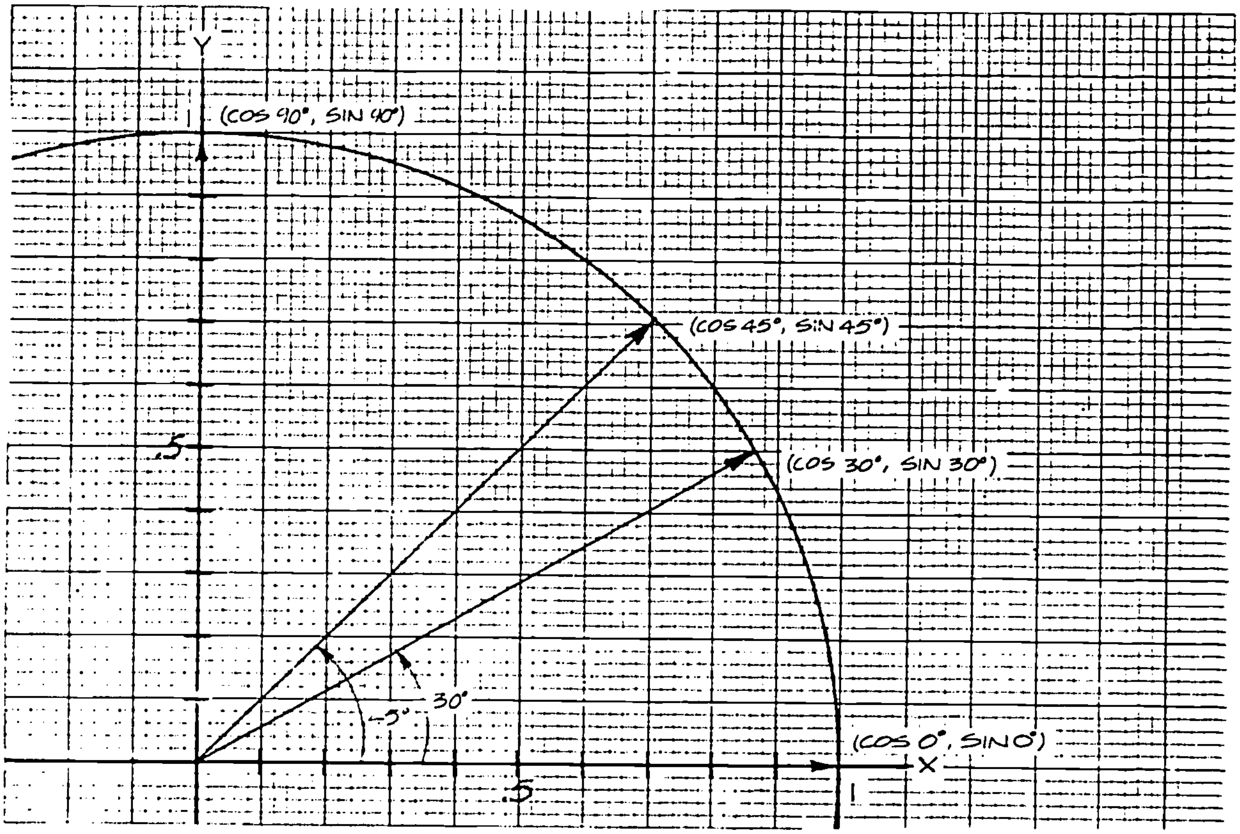


FIGURE 1

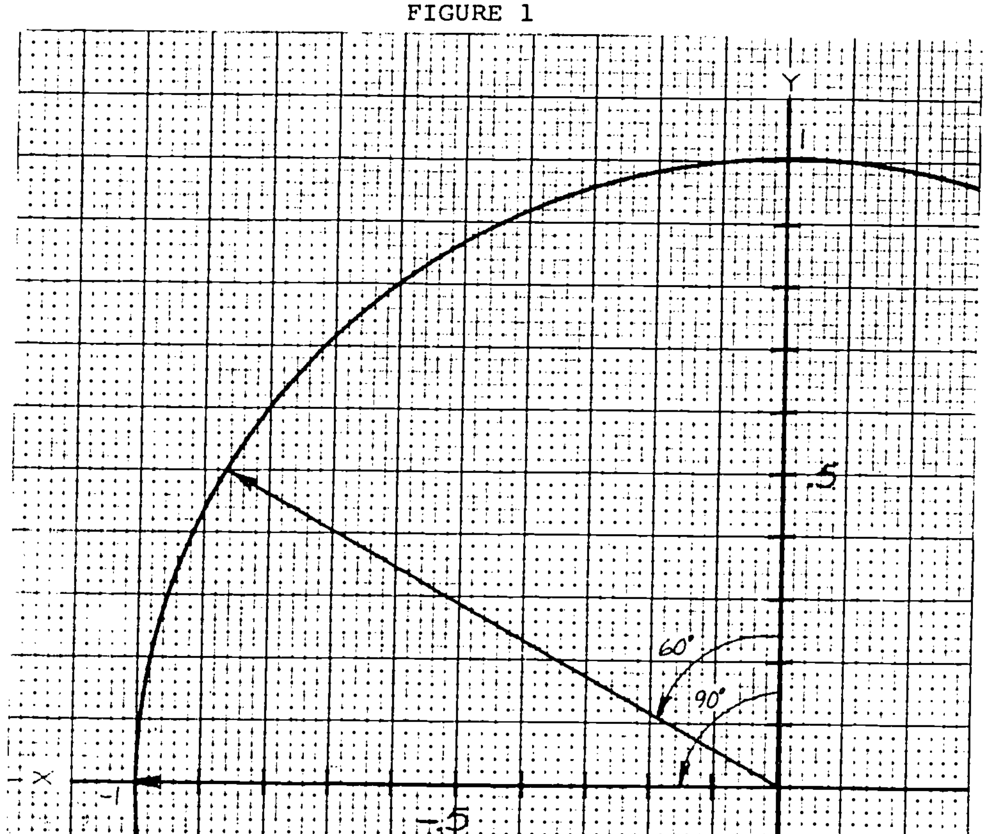


FIGURE 2

KEY--PROBLEM SET 17:

1. .94
2. .34
3. .94
4. -.64
5. -1
6. .94
7. 1
8. 1
9. x or horizontal
10. y or vertical
11.

	y	
sin is (+) cos is (-)	sin is (+) cos is (+)	
sin is (-) cos is (-)	sin is (-) cos is (+)	x

12. a. 1
    b. 0°
13. a. -1
    b. 180°
14. a. 1
    b. 90°
15. a. -1
    b. 270°
16. .88
17. .71
18. -.65
19. -.50
20. -.28
21. 0
22. -.71
23. .50
24. .28
25. -.35
26. .71
27. -.5
28. -.5
29. -.87
30. .71
31. 0
32. -1.0
33. 1.0
34. 1.0
35. -1.0
36. $\frac{\pi}{2}$ and $\frac{3\pi}{2}$
37. 0 and π
38. a. .79
    b. 2.36
    c. 3.93
    d. .52
    e. 2.62
    f. 3.67
    g. 1.05
    h. 2.09
    i. 3.14
    j. 4.19
    k. 5.24
    l. 6.28
39. a. 45°
    b. 135°
    c. 225°
    d. 30°
    e. 150°
    f. 210°
    g. 60°
    h. 120°
    i. 180°
    j. 240°
    k. 300°
    l. 360°

LESSON 18: A GRAPH OF y = sin x

<u>OBJECTIVES</u>:

The student will:

- construct a graph of y = sin x
- determine the value of cosine and sine for angles larger than 2π rad.

<u>PERIODS RECOMMENDED</u>: Two.

<u>OVERVIEW AND REMARKS</u>:

The x-axis in the graph of y = sin x is scaled in units of radians instead of π radians or degrees. The purposes of this complicating feature are threefold: (1) to reinforce the idea that π radians is a <u>number</u> of radians instead of some other entity; (2) to generate an undistorted graph of y = sin x; and (3) to lay the groundwork for the study of sound waves in which the units of radians (not π radians) occur naturally.

KEY--PROBLEM SET 18:

1. a.

x (rad)	x (degrees)	$y = \sin x$
0	0	0
.52	30	.50
.79	45	.71
1.00	57	.84
1.05	60	.87
1.50	86	1.00
1.57	90	1.00
2.00	115	.91
2.09	120	.87
2.36	135	.71
2.50	143	.60
2.62	150	.50
3.00	172	.14
3.14	180	.00

x (rad)	x (degrees)	$y = \sin x$
3.50	201	-.35
3.67	210	-.50
3.93	225	-.71
4.00	229	-.76
4.19	240	-.87
4.50	258	-.98
4.71	270	-1.00
5.00	286	-.96
5.24	300	-.87
5.50	315	-.71
5.76	330	-.50
6.00	344	-.28
6.28	360	.00

(1-b and 2 are on the following page.)

3. The next three angles in the given sequence are 1117°, 1477°, and 1837°. Also, any angle θ of the form

$$\theta = 37° \pm n360°$$

or $$\theta = 143° \pm n360°$$

where n is an integer, has the same sine as 37°.

4. The next three angles in the given sequence are $6\frac{1}{4}\pi$ rad, $8\frac{1}{4}\pi$ rad and $10\frac{1}{4}\pi$ rad. Also, any angle θ of the form

$$\theta = (\frac{\pi}{4} \pm 2n\pi) \text{ rad}$$

or $$\theta = (\frac{3\pi}{4} \pm 2n\pi) \text{ rad}$$

where n is an integer, has the same sine as $\frac{\pi}{4}$ rad.

5. The next three angles in the given sequence are 21.34 rad, 27.62 rad, and 33.90 rad. Also, any angle θ of the form

$$\theta = (2.5 \pm 2n\pi) \text{ rad}$$

or $$\theta = [(\pi - 2.5) \pm 2n\pi] \text{ rad}$$

where n is an integer, has the same sine as 2.5 rad.

6. a. $u = 2$, $v = \frac{\pi}{6}$

b. .5

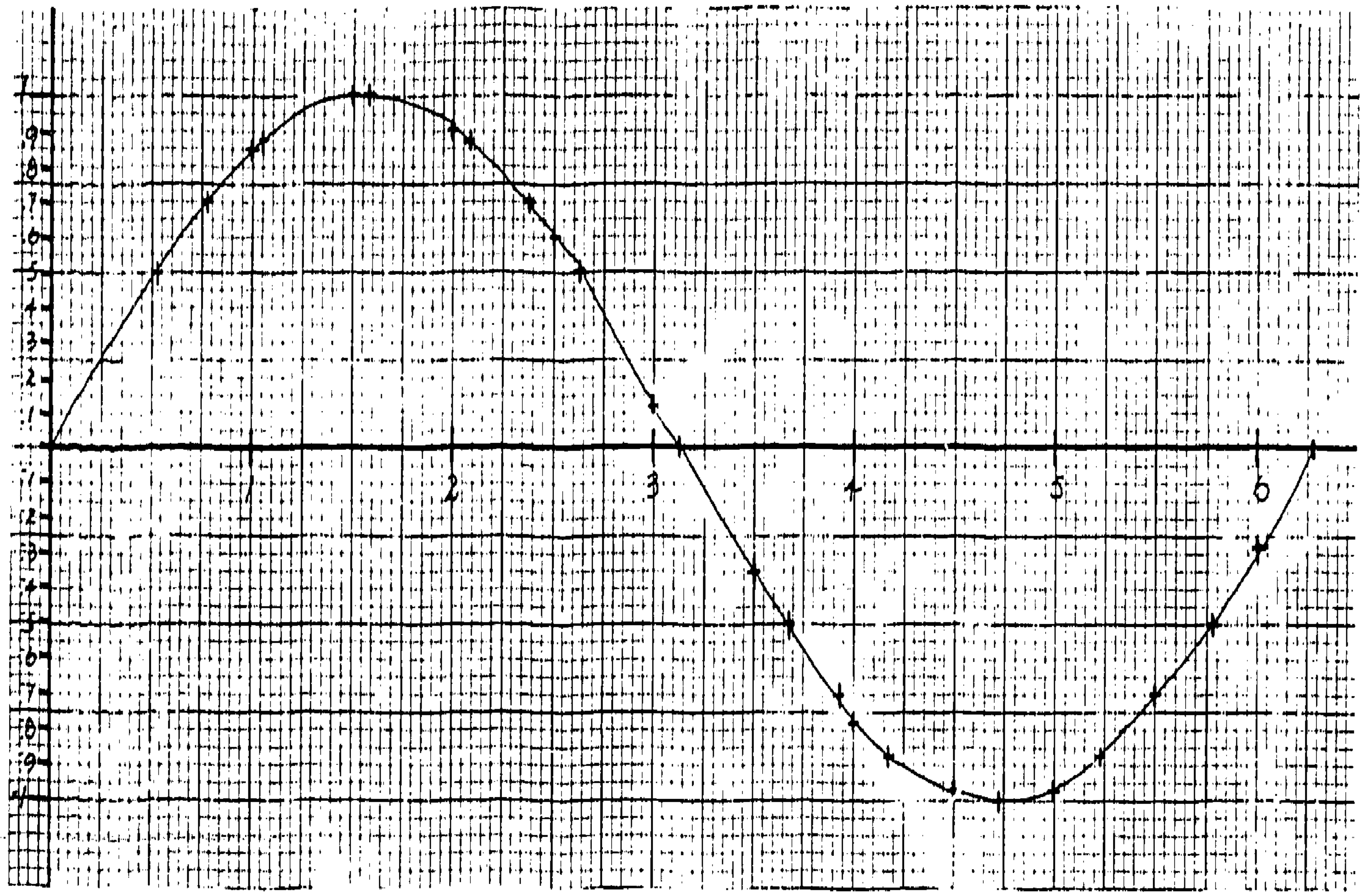
7. a. $u = 3$, $v = 3.5$

b. -.35

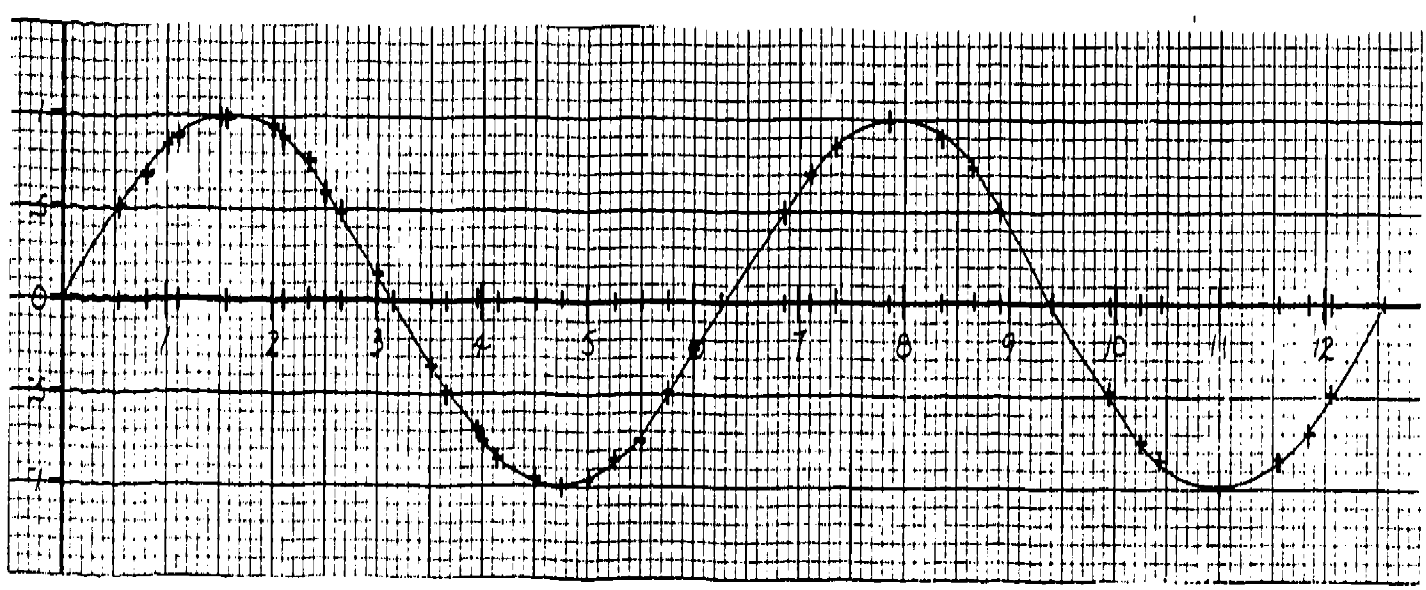
8. a. $u = 2$, $v = 200$

b. -.35

1. b.



2.



9. $\sin 540° = 0$

10. $\sin (7.78 \text{ rad}) \approx 1.00$

11. $\sin (17\frac{3}{4}\pi \text{ rad}) \approx -.71$

12. $\sin 806° \approx 1.00$

13. $\sin (20.84 \text{ rad}) \approx .91$

14. $\sin 1857 \approx .84$

15. $\sin (46\frac{1}{6}\pi \text{ rad}) = .5$

16. $\sin (23.84 \text{ rad}) \approx -.96$

17. $\sin 3630° = .5$

18. $\sin 360057° \approx .84$

19. $\sin (19\frac{1}{3}\pi \text{ rad}) \approx -.87$

20. $\sin (1001\frac{2}{3}\pi \text{ rad}) = -.87$

LESSON 19: TRIGONOMETRIC FUNCTIONS AND RIGHT TRIANGLES

OBJECTIVES:

The student will determine the values of sin, cos and tan from right triangles.

PERIODS RECOMMENDED: One.

SUPPLIES:

Transparency Masters IV-M-19a,b

OVERVIEW AND REMARKS:

You may wish to use the transparencies to introduce the new ideas of this lesson.

In conjunction with Transparency IV-M-19a, you may wish to have the students actually draw as large a 60°-30° triangle as possible and determine the appropriate ratios by measurement.

KEY--PROBLEM SET 19:

1. opposite

2. adjacent

3. opposite, adjacent

4. a. $\frac{4}{5}$
   b. $\frac{3}{5}$
   c. $\frac{4}{3}$

5. a. $2\sqrt{13}$
   b. 6
   c. 4
   d. $\frac{3\sqrt{13}}{13}$
   e. $\frac{2\sqrt{13}}{13}$
   f. $\frac{3}{2}$

6. a. $\frac{28}{53}$
   b. $\frac{45}{53}$
   c. $\frac{28}{45}$
   d. $\frac{45}{53}$
   e. $\frac{28}{53}$
   f. $\frac{45}{28}$

7. a. $c^2 = 3^2 + 5^2 = 34$, so $c = \sqrt{34}$
   b. $\frac{3\sqrt{34}}{34}$
   c. $\frac{5\sqrt{34}}{34}$

8. a. 1
   b. $c^2 = 1^2 + 1^2 = 2$, so $c = \sqrt{2}$
   c. $\frac{1}{\sqrt{2}}$
   d. $\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$
   e. $\frac{\sqrt{2}}{2}$

9. a. $\frac{1}{2}$
   b. $\frac{1}{2}$
   c. $b^2 = 2^2 - 1^2 = 3$, so $b = \sqrt{3}$
   d.

	30°	60°
$\sin\theta$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$
$\cos\theta$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$
$\tan\theta$	$\frac{\sqrt{3}}{3}$	$\sqrt{3}$

LESSON 20: THE USE OF TRIGONOMETRIC TABLES

OBJECTIVE:

The student will use the idea of a reference angle together with a table of trigonometric functions to find the values of trigonometric functions in the four quadrants.

PERIODS RECOMMENDED: One.

SUPPLIES:

Transparency Masters IV-M-20a,b

OVERVIEW AND REMARKS:

If you decide to use Transparency IV-M-20a to help introduce the idea of a reference angle, be sure to cut out with scissors the two triangles on the bottom before class commences. Then the cut-out triangles can be physically moved around on the circle.

KEY--PROBLEM SET 20:

1. .515
2. .191
3. 57.29
4. .423
5. .035
6. .616
7. a. II
   b. 180° - 122° = 58°
   c. .848
   d. .530
   e. positive
   f. negative
   g. .848
   h. -.530
8. a. 35°
   b. negative
   c. negative
   d. -.574
   e. -.819
9. a. 70°
   b. .342
   c. -.940
10. -.731
11. -.974
12. .788
13. -.122
14. -.208
15. .588
16. -.259
17. .988
18. .966
19. -.682
20. .974
21. -.946
22. .208
23. -.891
24. -.934
25. .018
26. -.961
27. .998
28. .788
29. -.766
30. -.985

LESSON 21: USING TRIGONOMETRIC FUNCTIONS TO SOLVE PROBLEMS

OBJECTIVE:

The student will determine the lengths of sides of right triangles by using trigonometric relationships.

PERIODS RECOMMENDED: One.

KEY--PROBLEM SET 21:

1. 64.3 mm
2. 663.5 cm
3. 65.8 m
4. 56.7 mm
5. 20.4 km
6. 180.9 m
7. 34.6 m
8. 143.7 cm
9. a. Only if $\angle\beta \approx 90°$ will $\cos 28° \approx \frac{A}{B}$
   b. $\angle\alpha \approx 15°$

10. a. $[141.4, 141.4]_{\text{rectangular}}$

b. $[29.55, 5.22]_{\text{rectangular}}$

c. $[170.95, 146.62]_{\text{rectangular}}$

d. $\tan \theta \approx .858$

e. $\theta \approx 41°$

f. Since θ is not exactly 41°, three slightly different answers may be obtained.

$\frac{146.62}{\sin 41°} \approx 223.5$ km/hr

$\frac{170.95}{\cos 41°} \approx 226.4$ km/hr

$\sqrt{(146.62)^2 + (170.95)^2} \approx 225.2$ km/hr

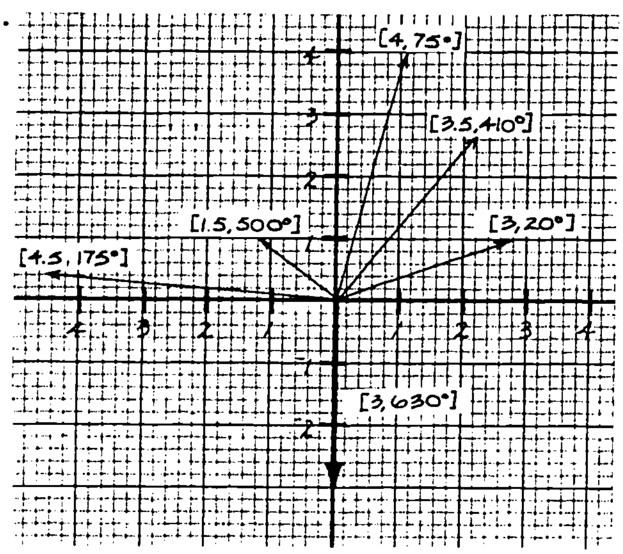
LESSON 22: REVIEW

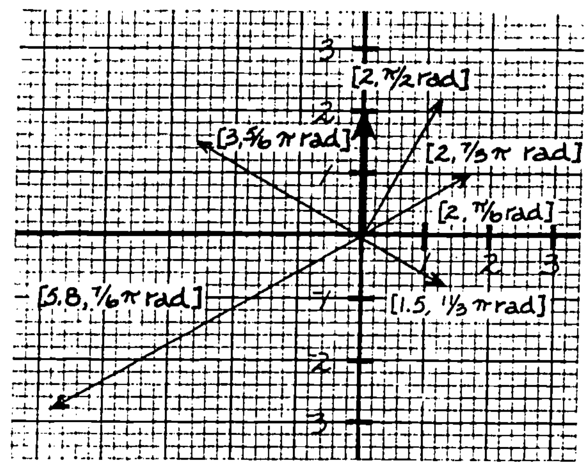
OBJECTIVE:

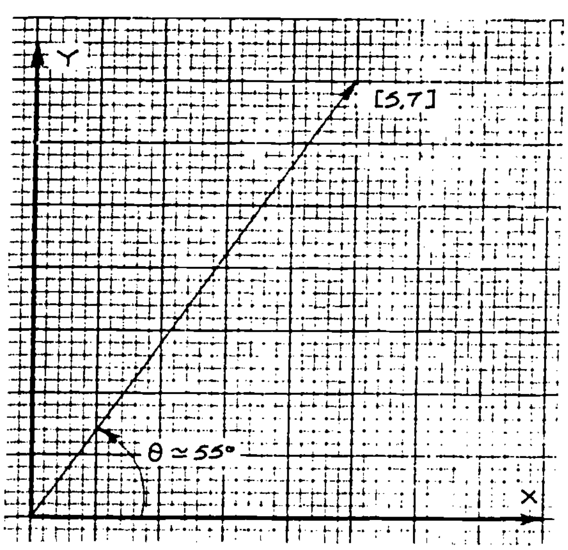
The student will solve problems related to the objectives of Lessons 10 through 21.

PERIODS RECOMMENDED: One or two.

KEY--REVIEW PROBLEM SET 22:

1. 

2. 

3. 

a. See graph.

b. See graph.

c. magnitude $= \sqrt{5^2 + 7^2} = \sqrt{74} \approx 8.6$

d. $\theta \approx 55°$

e. [8.6, 55°]

4. $\vec{R} \approx [-.9, 4.1]$ or $[4.2, 103°]$

5. $a = 3\pi$ rad $b = 4\pi$ rad $c = 5\frac{1}{2}\pi$ rad $d = 7\frac{1}{2}\pi$ rad $e = 9\pi$ rad

6. a. 3 cycles
   b. 1 cycle

7. a. I and II
   b. I and IV
   c. I

8. a. 105°
   b. 140°
   c. (-.91, .42)
   d. (-.91, -.42)
   e. 220°
   f. (-.26, -.97)
   g. 285°
   h. (.77, -.64)
   i. (.91, -.42)

9. a. $\sqrt{2}$
   b. $\frac{\sqrt{2}}{2}$
   c. $\frac{\sqrt{2}}{2}$
   d. 1

10. a. 2
    b. $\sqrt{3}$
    c. $\frac{\sqrt{3}}{2}$
    d. $\frac{\sqrt{3}}{2}$

11. a. $\frac{\sqrt{21}}{5}$
    b. $\frac{2}{5}$
    c. $\frac{\sqrt{21}}{2}$
    d. $\frac{2\sqrt{21}}{21}$
    e. $\frac{2}{5}$

12. a. .933
    b. .122
    c. .921
    d. 1.483

13. a. II
    b. positive
    c. 6°
    d. .105

14. -.946

15. -.875

16. -.707

17. -.342

18. 82 cm

19. 12 μm

20. 62 km

LESSON 23: AMPLITUDE

OBJECTIVES:

The student will:

- state the amplitude of given sine curves.
- state the amplitude of sine curves generated from specific equations.

PERIODS RECOMMENDED:

One

SUPPLIES AND EQUIPMENT:

1 24-gauge wire, about 10 cm long, 2 stripped ends

5 24-gauge wires, about 40 cm long, 2 stripped ends (wire can be obtained from the Science teacher)

4 alligator clips

BIP

Oscilloscope

Speaker (Permanent Magnet Type, 4 to 16 Ω)

51-ohm resistor (color code green, brown, black, and silver or gold)

OVERVIEW AND REMARKS:

This is the first lesson in an interdisciplinary sequence on sound and hearing. The mathematics portion of the curriculum will develop the vocabulary associated with sound, i.e., amplitude, frequency, wavelength, beats and harmonics. This will be done with the aid of teacher demonstrations with the oscilloscope. Science will use the vocabulary and concepts developed in Mathematics to determine the speed of sound experimentally.

It will be essential for you and your Biomedical Science colleague to coordinate efforts. Close timing is important so that students will use concepts in the Science course shortly after they learn them in the Mathematics course. Also, the Science teacher will need to be familiar with the material in Mathematics Lessons 23-29. Some team teaching in this sequence is an attractive possibility.

Laboratory Activity 21 is intended to reinforce the basic wave concepts introduced in Mathematics Lessons 23-25, i.e., amplitude, frequency, period, wavelength and speed of a traveling wave. In the Laboratory Activity the students use a "slinky" or some equivalent device to generate standing waves. The frequency and wavelength of the waves may be determined by direct observation and the speed of propagation may then be calculated.

You should arrange your relative pacing with the Science instructor so that Mathematics Lesson 25 is taught just <u>before</u> Laboratory Activity 21. However, the slinky part of the activity is one of the "floating" variety. If you are too far ahead of Science, you might consider the possibility of teaching this particular activity in Mathematics after Section 25 is completed. The materials required are not elaborate, i.e., some stopwatches, some slinkies and some tape.

The most important point to keep in mind is that the Science sequence on sound (beginning with Lesson 21) cannot be taught until Mathematics Lesson 25 has been presented.

TEACHER ACTIVITIES:

During this class you will be demonstrating how the amplitude of the image of a pure tone varies with the loudness of the sound. If you are unfamiliar with the operation of the BIP or or an oscilloscope, refer to the section "Oscilloscope and BIP Directions" which follows.

After the presentation of the material in the text, you may want to have the students begin working on the problem set. Then small groups of students can be allowed to work with the volume control in order to double the amplitude, halve the amplitude and so forth. You may wish to connect the 51-ohm resistor in series with the speaker in order to cut down the maximum possible volume to a tolerable level.

CAUTION: Do not plug the resistor or capacitor leads directly into the programming terminals on the BIP. The resistor wires are larger than the terminals are designed to accept, although they may be forced in. However, if they are forced in, the terminal will be sprung and will no longer make good contact with the size of wire for which it was designed. Sprung terminals must be replaced.

To connect the resistor in the circuit attach alligator clips to each end and attach one wire to the speaker and one wire to the oscilloscope.

OSCILLOSCOPE AND BIP DIRECTIONS:

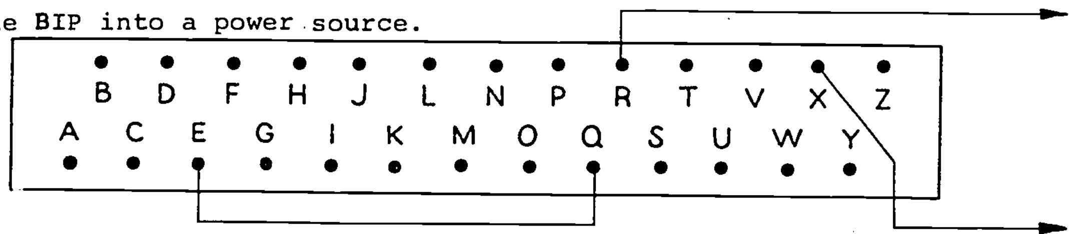
A. Programming the BIP.

1. Connect E to Q.

2. Plug one end of a wire (40 cm) into terminal R.

3. Plug one end of another long wire into terminal.

4. Plug the BIP into a power source.



B. Connect the BIP and speaker to the oscilloscope.

1. Connect the BIP to the oscilloscope as follows.

a. Connect the wire attached to terminal X to the oscilloscope ground. The ground terminal is usually denoted by the symbol ⏚ or "GND."

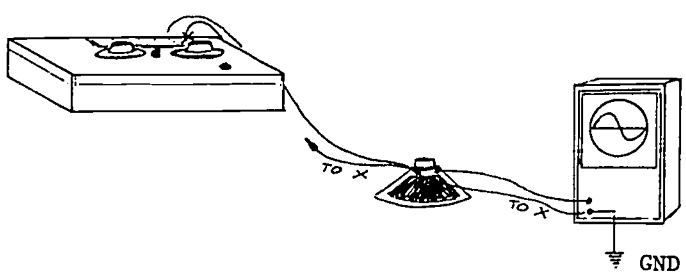
b. Connect the wire attached to terminal R to the vertical input of the scope. Various abbreviations are used on different scopes to signify this. Among them are "v-input" and "vert. in." At any rate, it is usually found above one of the ground terminals (most oscilloscopes have two).

2. Connect the speaker to the oscilloscope.

a. Connect an alligator clip and wire to each terminal of the speaker.

b. Connect one of the wires to the vertical input and the other to the ground terminal.

3. The final hook-up should be as shown in the diagram below.



4. Experiment with the amplifier gain control and frequency control dials and the frequency-range switch to familiarize yourself with their effects.

C. Adjusting the scope.

1. a. Turn on the scope.

b. Turn up the intensity control.

2. If there is a "60∿" switch on the scope control panel, make sure that you do <u>not</u> have it switched on.

3. a. Locate the knobs that control the location of the display on the screen. These may be called "V-pos" and "H-pos" or "vert. cen." and "hor. cen." The vertical-centering knob will move the display up or down. The horizontal-position knob will move the display to the right and left. Experiment with the knobs until you locate them.

b. Adjust the two controls until the display is more or less in the center of the screen.

c. If all you get is a dot in the middle of the screen, then proceed immediately to Step 5. A dot will sometimes burn the screen. Step 5 will tell you how to turn a dot into a horizontal line.

4. a. Locate the knobs that control the vertical dimensions of the display. Generally there are both coarse and fine controls for this. Again, the labels for these controls vary from scope to scope. You might find them labeled "Vert-Gain," "Vert. Attenuator," or simply "Vertical." You will know you have found the correct controls when you can vary the height of the display.

b. Adjust the display until it fills approximately half the vertical dimensions of the screen.

5. a. Locate the controls for the horizontal dimensions. The horizontal controls will be analogous to the vertical controls in the physical layout of the control panel, i.e., vertical on the right implies horizontal on the left. You will know you have located the right knob or knobs when you can vary the width of the display.

b. Adjust the width of the display until it just fills the screen.

6. a. In order to "stop" a sine wave, you will have to adjust the horizontal sweep frequency. For example, if the electron beam goes from left to right during one sound-wave cycle, then you will see a sine curve on the screen.

Again, there are generally coarse and fine controls for the horizontal sweep frequencies. They might be labeled "sweep selector (coarse control)" and "sweep vernier (fine control)." Or both controls may be combined under the cryptic label "frequency." You will know you have found the right controls when you can vary the pattern of movement across the screen.

b. Adjust the sweep frequency until you have $1\frac{1}{2}$ to $2\frac{1}{2}$ complete cycles shown on the screen. You may not be able to stop the wave completely. This is the subject of the next step.

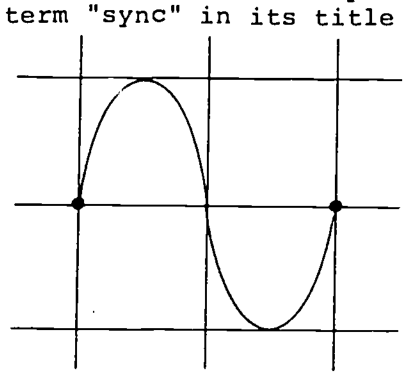
7. a. Since most electronic equipment is unstable, oscilloscopes have circuits that will stabilize the waveforms displayed on the screen. These circuits are called "synchronizer" circuits. Depending upon chance, you may or may not have this circuit switched in. If the sine wave won't stop, no matter how carefully you adjust the sweep controls, you then know the synchronizer circuit is not switched in.

In this case, go on to Part b; otherwise, omit it.

b. Experiment with the remainder of the controls until you succeed in stopping the display. Any knob that has the term "sync" in its title would be a likely candidate.

8. If you have successfully arrived at this point, you should be prepared to perform the demonstrations to follow.

D. Adjust the scope so that the displayed sine wave has an amplitude of one and a period of two. The display will look like the figure to the right when this adjustment has been made.



E. Notice that the amplitude and the loudness may be varied simultaneously by turning the amplifier gain control (left dial) of the BIP.

KEY--PROBLEM SET 23:

1. 400	5. 50	9. 10	13. 2	17. 10^{-7}	21. 5
2. 13	6. 3	10. 15	14. a	18. 432	22. 100
3. 50	7. 20	11. 15	15. π	19. .07	
4. .05	8. 300	12. $\frac{2}{5}$	16. 7	20. 14×10^{24}	

LESSON 24: FREQUENCY AND PERIOD

OBJECTIVES:

The student will:

- identify the frequency of given sine curves.
- identify the frequency associated with equations of the form $y = \sin bt$.
- use the words frequency and period correctly.

SUPPLIES AND EQUIPMENT:

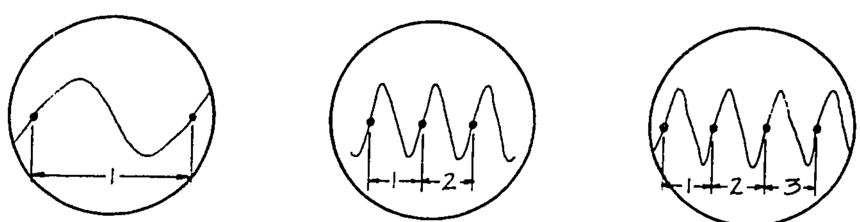
The same demonstration equipment used in Lesson 23.

TEACHER ACTIVITIES:

Conduct a preliminary discussion on sound and its representation as a sine wave. Review amplitude by repeating the oscilloscope demonstration of Lesson 23. Stress the point that by increasing the value of a in the equation $y = a \sin t$ one increases the amplitude of the sine curve. Further, review that amplitude is related to loudness. Extend the demonstration by varying the pitch of the note. Obtain a single, stable sine wave on the oscilloscope using a convenient frequency, then proceed to relate pitch to frequency in the following manner.

Double the frequency output of the signal generator and observe that two sine waves are now displayed in the space that used to contain one.

Increase the frequency by a factor of three and observe three complete sine waves, etc. See Figure 1.



2 FREQUENCY 1 2 (FREQUENCY 1) 3(FREQUENCY 1)

Observe that in order to get a stable screen image one must use whole multiples of the original frequency. The ratio of the period of the oscilloscope sweep to the period of the input signal must be a whole number. Otherwise, the display of each single sweep will be out of phase with each succeeding sweep.

Establish with the class that an increase in pitch "squeezes" the waves together like an accordion. Explain that the topic for this lesson will be how to stretch and squeeze the sine curve mathematically.

Once again, you may wish to leave the demonstration apparatus set up and involve the students in knob adjustment in a one-on-one situation. You could invite the students up one at a time and ask them to double, triple or halve the frequency of the displayed curve by changing the frequency-control dial on the BIP.

The remainder of the class can work on the problem set.

KEY--PROBLEM SET 24:

1. a. T
   b. f
   c. T
   d. T
   e. f
   f. T

2. $f = \frac{1}{7}\ \frac{\text{cycle}}{\text{hour}}$

   $T = \frac{7\ \text{hours}}{\text{cycle}}$

3. $f = \frac{1}{\pi}\ \frac{\text{cycle}}{\text{hour}}$

   $T = \pi\ \frac{\text{hours}}{\text{cycle}}$

4. $f = \frac{1\ \text{cycle}}{.3\ \text{sec}} = 3\frac{1}{3}\ \frac{\text{cycles}}{\text{sec}}$

   $T = .3\ \frac{\text{sec}}{\text{cycle}}$

5. $f = 1\ \frac{\text{cycle}}{\text{week}}$

   $T = 1\ \frac{\text{week}}{\text{cycle}}$

6. $f = \frac{5\ \text{cycles}}{10\ \text{days}}$

   $= \frac{1}{2}\ \frac{\text{cycle}}{\text{day}}$

   $T = 2\ \frac{\text{days}}{\text{cycle}}$

7. $f = \frac{\frac{1}{2}\ \text{cycle}}{1\ \mu\text{sec}}$

   $= \frac{1}{2}\ \frac{\text{cycle}}{\mu\text{sec}} =$

   $T = 2\ \frac{\mu\text{sec}}{\text{cycle}}$

8. $f = \frac{.25\ \text{cycle}}{.5\ \text{sec}}$

   $= \frac{1}{2}\ \frac{\text{cycle}}{\text{sec}}$

   $T = 2\ \frac{\text{sec}}{\text{cycle}}$

9. $f = \frac{\frac{3}{2}\ \text{cycles}}{3\ \text{sec}}$

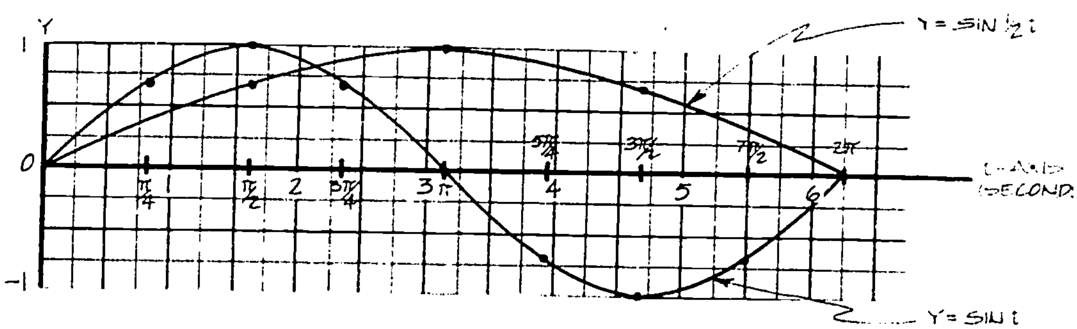
   $= \frac{1}{2}\ \frac{\text{cycle}}{\text{sec}}$

   $T = 2\ \frac{\text{sec}}{\text{cycle}}$

10. a.

t (sec)	sin t	sin $\frac{1}{2}$t
0	0	0
$\frac{\pi}{4}$	.71	SKIP
$\frac{\pi}{2}$	1	.71
$\frac{3\pi}{4}$	.71	SKIP
π	0	1
$\frac{5\pi}{4}$	-.71	SKIP
$\frac{3\pi}{2}$	-1	.71
$\frac{7\pi}{4}$	-.71	SKIP
2π	0	0

b.



10. c. $y = \sin t$

$f = \frac{1}{2\pi} \frac{\text{cycle}}{\text{sec}}$ $T = 2\pi \frac{\text{sec}}{\text{cycle}}$

$y = \sin \frac{1}{2}t$

$f = \frac{\frac{1}{2}}{2\pi} \frac{\text{cycle}}{\text{sec}} = \frac{1}{4\pi} \frac{\text{cycle}}{\text{sec}}$

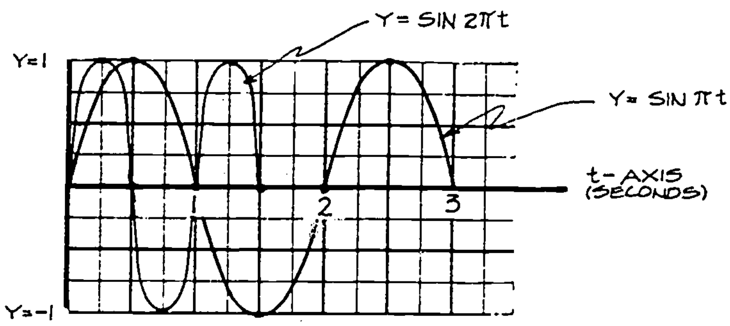
$T = 4\pi \frac{\text{sec}}{\text{cycle}}$

11. a.

t (sec)	sin πt
0	0
.5	1
1.0	0
1.5	-1
2.0	0
2.5	1
3.0	0

t (sec)	sin 2πt
0	0
.25	1
.50	0
.75	-1
1.00	0
1.25	1
1.50	0

b.



c. $y = \sin \pi t$

$f = \frac{\pi \text{ cycles}}{2\pi \text{ sec}}$

$= \frac{1}{2} \frac{\text{cycle}}{\text{sec}}$

$T = 2 \frac{\text{sec}}{\text{cycle}}$

$y = \sin 2\pi t$

$f = \frac{2\pi \text{ cycles}}{2\pi \text{ sec}}$

$= 1 \frac{\text{cycle}}{\text{sec}}$.

$T = 1 \frac{\text{sec}}{\text{cycle}}$

12. $f = \frac{1}{2\pi} \frac{\text{cycle}}{\text{sec}}$

$T = 2\pi \frac{\text{sec}}{\text{cycle}}$

13. $f = \frac{33}{2\pi} \frac{\text{cycles}}{\text{sec}}$

$T = \frac{2\pi}{33} \frac{\text{sec}}{\text{cycle}}$

14. $f = \frac{1}{2} \frac{\text{cycle}}{\text{sec}}$

$T = 2 \frac{\text{sec}}{\text{cycle}}$

15. $f = 8 \frac{\text{cycles}}{\text{sec}}$

$T = \frac{1}{8} \frac{\text{sec}}{\text{cycle}}$

16. $f = \frac{3}{2} \frac{\text{cycles}}{\text{sec}}$

$T = \frac{2}{3} \frac{\text{sec}}{\text{cycle}}$

17. $f = \frac{1}{4\pi} \frac{\text{cycle}}{\text{sec}}$

$T = 4\pi \frac{\text{sec}}{\text{cycle}}$

18. $y = \sin 16\pi t$

LESSON 25: TRAVELING WAVES AND WAVELENGTH

OBJECTIVES:

The student will:

- determine the wavelengths of given sine curves and equations.
- state whether wavelength or period (or frequency) is appropriate to describe a given sine curve when given the units of the horizontal axis.
- use the equation (frequency) · (wavelength) = (speed of wave) to find one of the variables when the other two are given.

PERIODS RECOMMENDED: One or two.

SUPPLIES:

Transparency Master IV-M-25

OVERVIEW AND REMARKS:

The transparency can be used to present another example like the one at the end of the section.

KEY--PROBLEM SET 25:

1. a. λ
   b. T
   c. λ
   d. f
2. a. f
   b. λ
   c. T
   d. T
   e. λ
   f. f
   g. λ

3. $\lambda = 7\ \frac{\text{meters}}{\text{cycle}}$

4. $T = 2\ \frac{\text{sec}}{\text{cycle}}$

5. $\lambda = 10\ \frac{\text{cm}}{\text{cycle}}$

6. $\lambda = 10\ \frac{\text{mm}}{\text{cycle}}$

7. $T = 1\ \frac{\text{hour}}{\text{cycle}}$

8. $2\pi\ \frac{\text{meters}}{\text{cycle}}$

9. $\frac{\pi}{7}\ \frac{\text{meter}}{\text{cycle}}$

10. $2\pi^2\ \frac{\text{meters}}{\text{cycle}}$

11. $\frac{2}{3}\ \frac{\text{meter}}{\text{cycle}}$

12. $42{,}000\ \frac{\text{m}}{\text{sec}}$

13. $48 \times 10^4\ \frac{\text{cm}}{\text{sec}}$

14. $3 \times 10^3\ \frac{\text{mm}}{\text{sec}}$

LESSON 26: ADDING WAVES AND PHASE

OBJECTIVE:

The student will construct graphs of functions of the form $y = \sin t + \sin\left(t + \frac{n\pi}{8}\right)$ where n is an integer between 0 and 8. Two methods will be employed, one numerical and one graphical.

PERIODS RECOMMENDED: One or two.

SUPPLIES:

Transparency Master IV-M-26

Compasses (1 per student)

graph paper (2 sheets per student)

OVERVIEW AND REMARKS:

The lessons so far have all been concerned with pure tones. However, most sounds are made up of a combination of tones. In this lesson, and the two that follow, the students will examine what happens when two tones are combined. This is equivalent to studying the waveforms that result from the addition of two sine waves. In the Science activity involving the slinky, the students will see how the combination of incident and reflected waves produce a standing wave. Also, the useful ideas of beats and harmonics will be uncovered by a patient study of the addition of waves.

How the material is organized to investigate the nature of wave addition is outlined below.

Lesson Number	
26	Students graph $y = \sin t + \sin (t + \frac{n\pi}{8})$ and observe the pattern of the graphs as n increases from 0 to 8 in steps of 1.
27	Students construct a set of graphs. On the graphs will be functions of the form $y = \sin nt + \sin mt$. Several different values of m and n will be used.
28	Beats and harmonics will be displayed on an oscilloscope. The class should be able to identify and describe the different patterns by this time.

TEACHER ACTIVITIES:

Phase is the subject of this lesson. The sine curves discussed in this lesson all have the same period, namely 2π. At some point during the period you will need to discuss the following definition of phase.

The phase, p, is the absolute value of the difference in horizontal coordinates of two adjacent peaks of the two waves to be added. If the peaks coincide ($p = 0$), the waves are said to be in phase. Otherwise they are said to be p units out of phase.

In this lesson each student will prepare a graph of the sum of two sine waves that are either in phase or out of phase by a multiple of $\frac{\pi}{8}$. It would be a good idea to have the graphing done in class so that you can help those students who are confused about the graphing methods.

Transparency IV-M-26 may be used to describe the two different methods we will use to add sin t and $\sin(t + \frac{n\pi}{8})$. The specific case in which $n = 3$ is dealt with in the transparency. All other values of n may be handled similarly.

The two methods we will use to add sin t and $\sin (t + \frac{n\pi}{8})$ are described on the next page.

1. Compass method. Measure the height at some value of t and mark it off on the bottom graph. Measure the remaining height and mark it off from the previous point. Note that each individual measurement may be either positive or negative. It is conceivable that students might become confused about how to handle this addition. Don't give up. The students probably will prefer it to the analytic method, which is also presented.

2. Analytic method. The algebraic sum of the two y-coordinates will give $[\sin t + \sin(t + \frac{n\pi}{8})]$. This sum can be plotted directly. This method is more accurate than the first method; however, it is much slower. It also lacks intuitive feel.

The sum of the two functions shown on the transparency should look like Figure 1 when plotted on the blank graph at the bottom of Transparency IV-M-26.

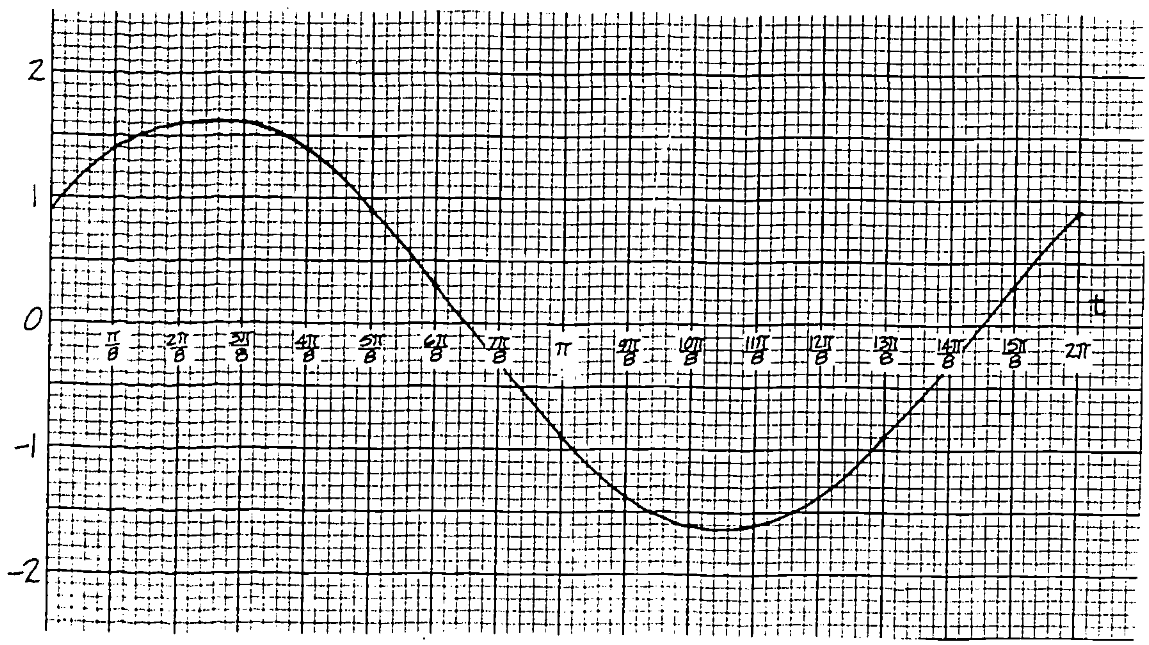
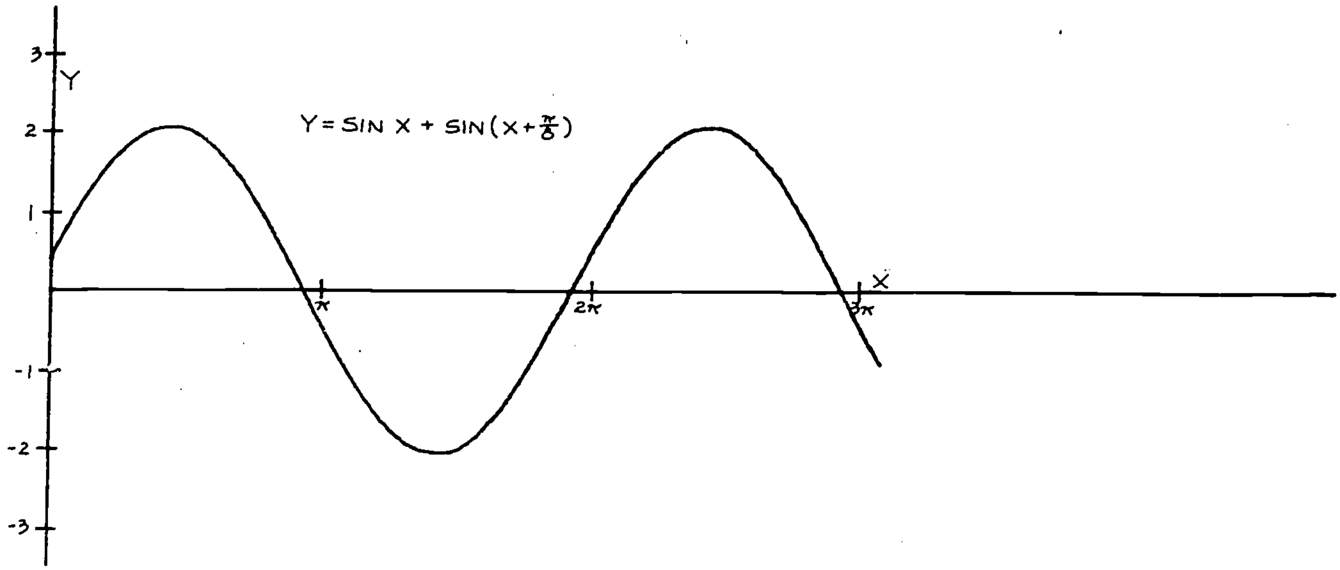


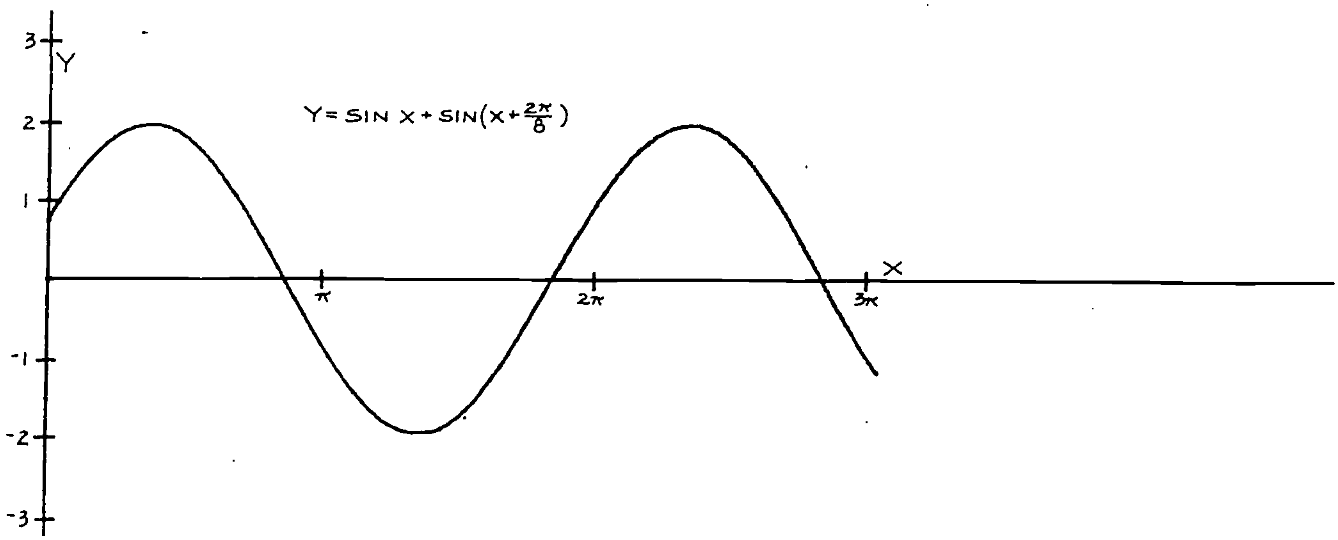
Figure 1

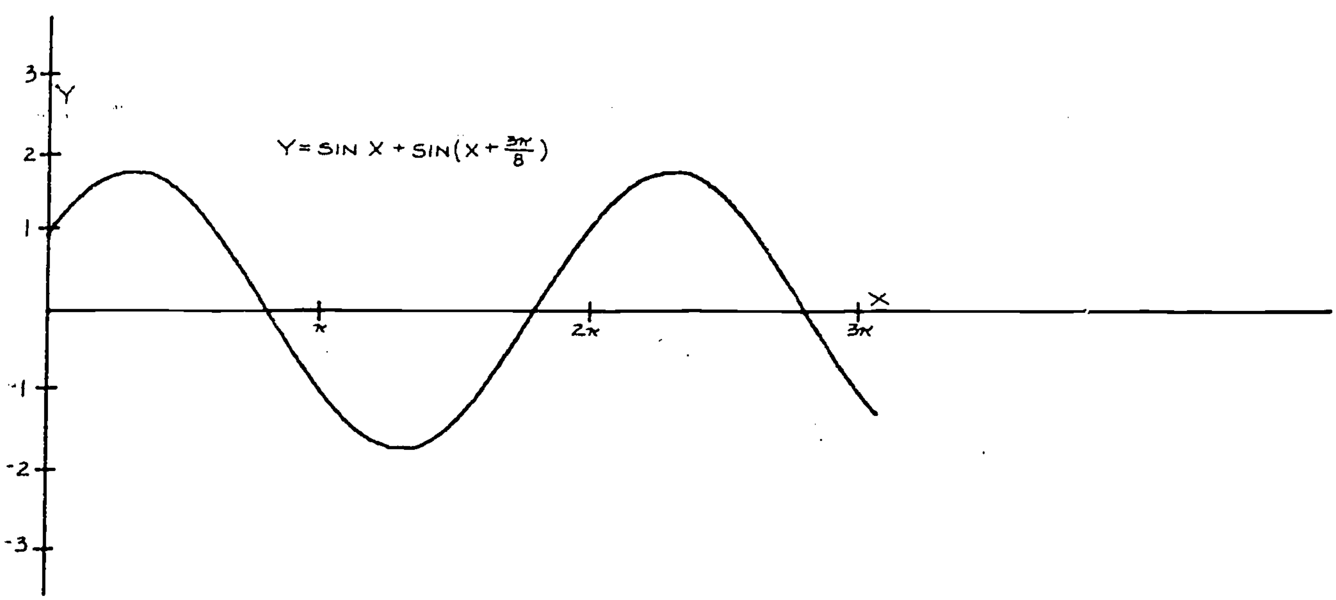
When you have satisfied yourself that the class understands, divide the class into nine work committees. Assign a different value of n from 0 to 8 to each committee. Since the whole set is to be displayed, the best effort of each committee should be selected for membership in the display set.

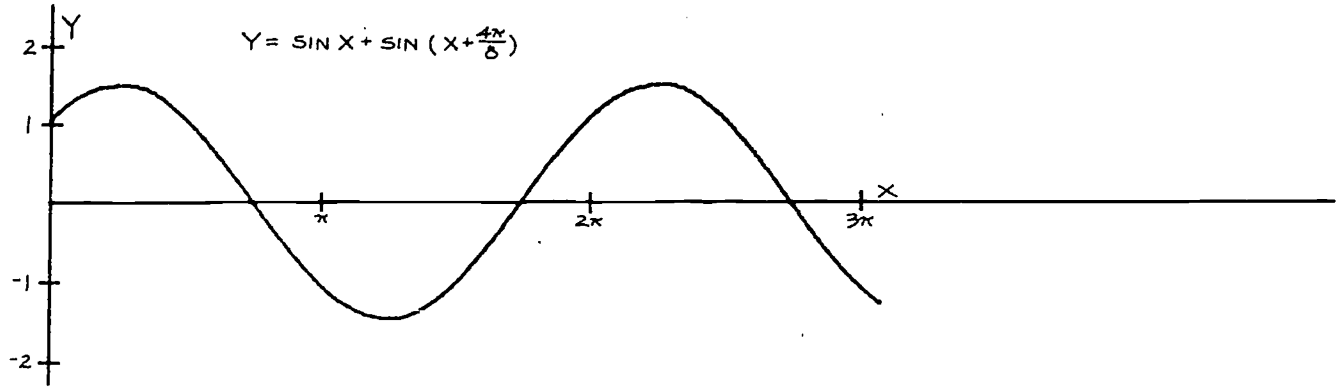
KEY--PROBLEM SET 26:

On the next page you will find the graphs of the functions $y = \sin t + \sin(t + \frac{n\pi}{8})$ for $1 \leq n \leq 7$. The graphs when $n = 0$ and $n = 8$ are omitted since they have a simple form. When $n = 0$ the function is $y = 2 \sin t$ and when $n = 8$ the function is equivalent to $y = 0$.

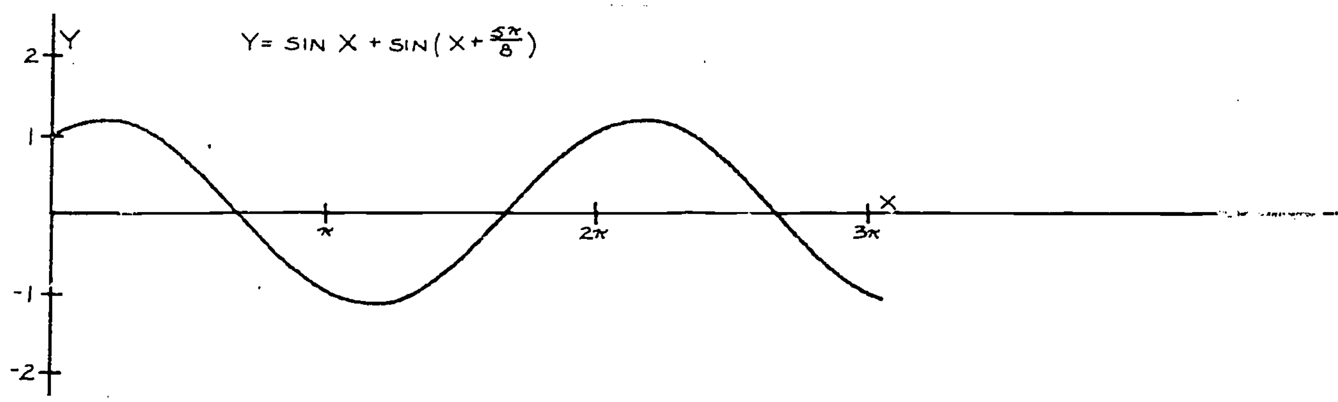

Y = SIN X + SIN(X + π/8)
Y
X
π
2π
3π
3
2
1
-1
-2
-3


Y = SIN X + SIN(X + 2π/8)
Y
X
π
2π
3π
3
2
1
-1
-2
-3

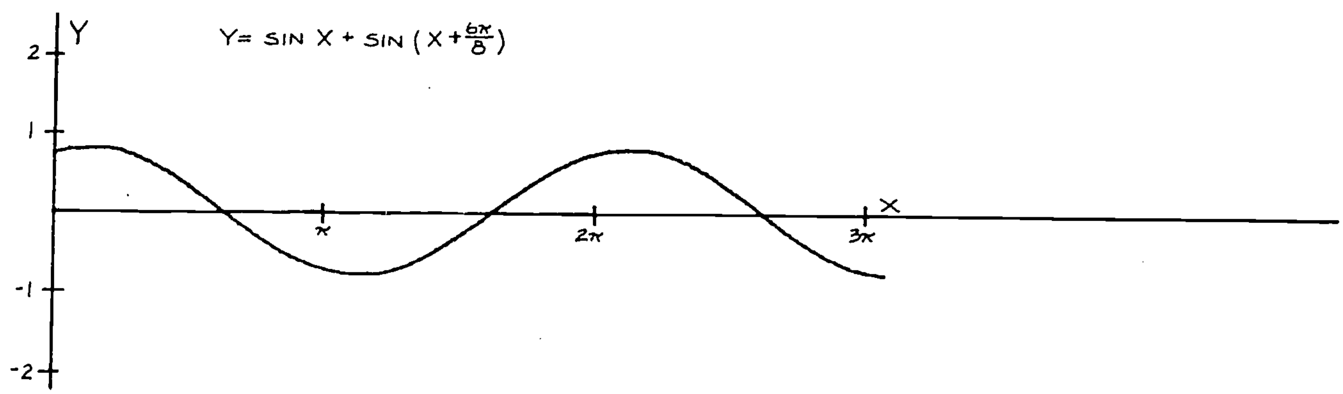

Y = SIN X + SIN(X + 3π/8)
Y
X
π
2π
3π
3
2
1
-1
-2
-3



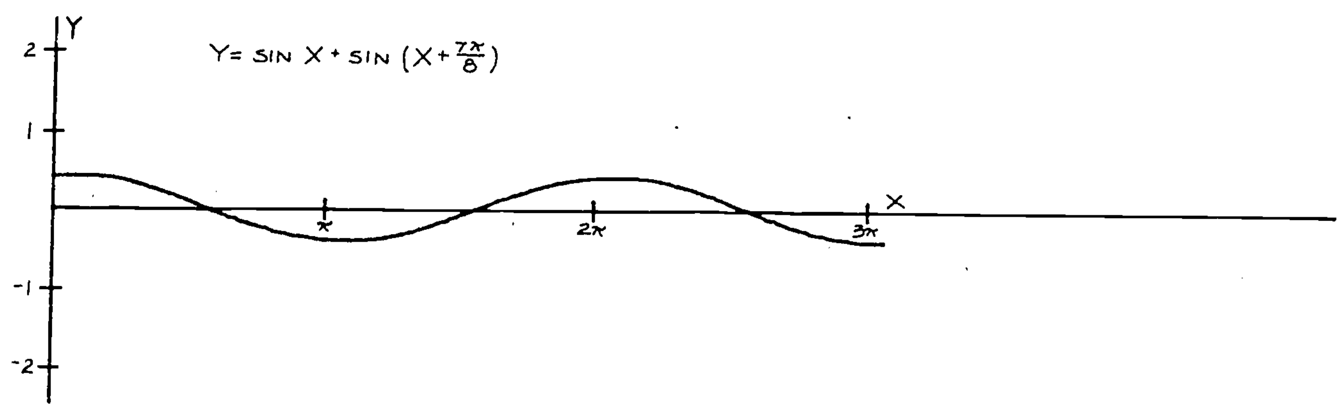
Y= SIN X + SIN (X+4π/8)
Y
X
2
1
-1
-2
π
2π
3π



Y= SIN X + SIN(X+5π/8)
Y
X
2
1
-1
-2
π
2π
3π



Y= SIN X + SIN (X+6π/8)
Y
X
2
1
-1
-2
π
2π
3π



Y= SIN X + SIN (X+7π/8)
Y
X
2
1
-1
-2
π
2π
3π

LESSON 27: ADDITION OF WAVES, $y = \sin mt + \sin nt$

OBJECTIVES:

Students will graph functions of the form $y = \sin mt + \sin nt$ where m and n $\varepsilon \{1, 2, 4, 8, 9\}$.

PERIODS RECOMMENDED: One or two.

SUPPLIES:

graph paper (1 sheet per student)

compasses (1 per student)

TEACHER ACTIVITIES:

A. Examine the student graphs from the last period. Select a good graph from each group. Display the complete set prominently in a sequential order. Make these observations.

1. $y = \sin t + \sin (t + \frac{n\pi}{8})$

when $n = 0$, $y = \sin t + \sin (t + 0)$

$$y = \sin t + \sin t$$

$$y = 2 \sin t$$

When this condition exists between two waves, they are said to be completely "in phase."

2. When $n = 8$, $y = \sin t + \sin (t + \pi)$.

Point out that the effect of adding π to t in $\sin (t + \pi)$ is to shift the entire sine curve π units to the left. When this happens, $\sin (t + \pi)$ is 0 when $t = 0$ and negative when $0 < t < \pi$, while sin t is positive in the same interval. Finally, $\sin (t + \pi) = -\sin t$. Substitute some numerical values of t, if necessary, to convince the class of the equality. Going on, we have

$$y = \sin t - \sin t$$

$$y = 0$$

When this condition exists between two waves, they are said to be completely "out of phase."

3. As n increases from 0 to 8, the amplitude of the sum decreases from 2 to 0. The term $\frac{n\pi}{8}$ specifies how far out of phase the two waves are. In fact, when $n = 3$ the two waves are said to be "$\frac{3}{8}\pi$ radians out of phase," or "67.5° out of phase."

4. Recall the definition of phase. Phase is the absolute value of the difference in horizontal coordinates of two adjacent maxima of the two waves to be added.

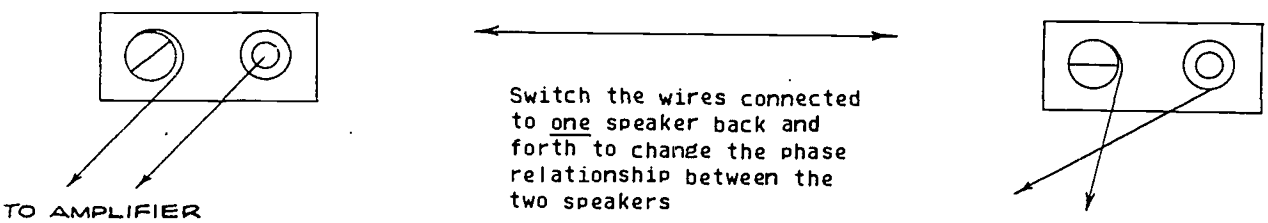
5. The period or wavelength of the sum is the same as that of sin t, i.e., 2π.

6. The peaks of the sum lie halfway between the peaks of the two added waves. The formula below is mainly for your information. Its purpose is to convince you of the generality of the first statement.

$$\sin t + \sin (t + k) = 2 \cos \frac{k}{2} \sin (t + \frac{k}{2})$$

Note the "+k" term on the left and the "$+\frac{k}{2}$" term on the right. $\frac{k}{2}$ is half the sum of zero and k. The implication is that the peaks and zeros of the sum will lie halfway between the peaks and zeros of the two added waves. To see this, first note that 2 cos $(\frac{k}{2})$ is constant. Sin t is zero when $t = n\pi$ and sin $(t + k)$ is zero when $t = -k + n\pi$. Sin $(t + \frac{k}{2})$ is zero when $t = -\frac{k}{2} + n\pi$.

While you are on the subject of phase, you may wish to mention an application. When hooking up stereo speakers, it is important to keep phase in mind. Speaker cones produce sound waves by vibrating in and out. Suppose one speaker cone goes in while the other one goes out. In this case, one speaker produces a rarefaction while the other produces a compression of the air. The tendency is for the two speakers to cancel out one another. This never happens completely because the two speakers do not occupy the same point in space. The objective when attempting to phase speakers is to get the speaker cones to move in tandem, i.e., to get them to move in and out together instead of in a syncopated manner. The relative phase of two speakers may be changed by reversing the contacts of the wires connected to one speaker as shown below. Switching the wires causes a 180° change in phase. People are generally surprised when they find that they can hear the difference.



B. In Problem Set 27 there is a set of graphs of the form y = sin nt for n ε {1, 2, 4, 8, 9}. Obtain a set of graphs of the form y = sin mt + sin nt in the following manner.

1. Divide the class into four to six groups. Make sure that there is at least one reliable student in each group.

2. Assign a pair of values of m and n to each group. The following pairs should definitely be included, in order to exemplify harmonics and beats.

m	n	
1	2	Addition of wave and its first harmonic.
2	4	
4	8	
8	9	beats

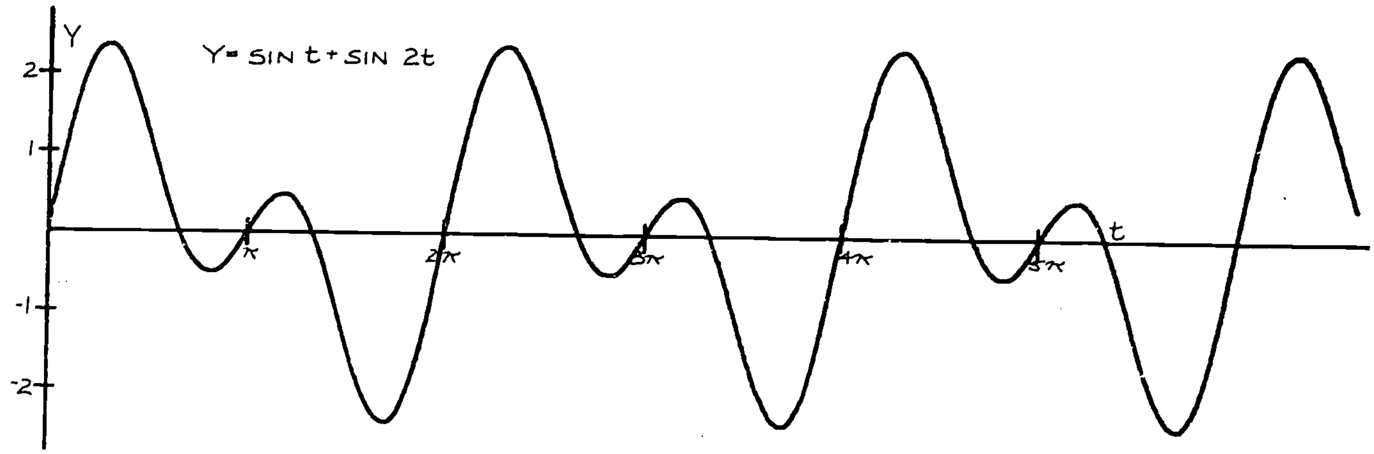
If you wish, you can include the following pairs in order to exemplify second and third harmonics.

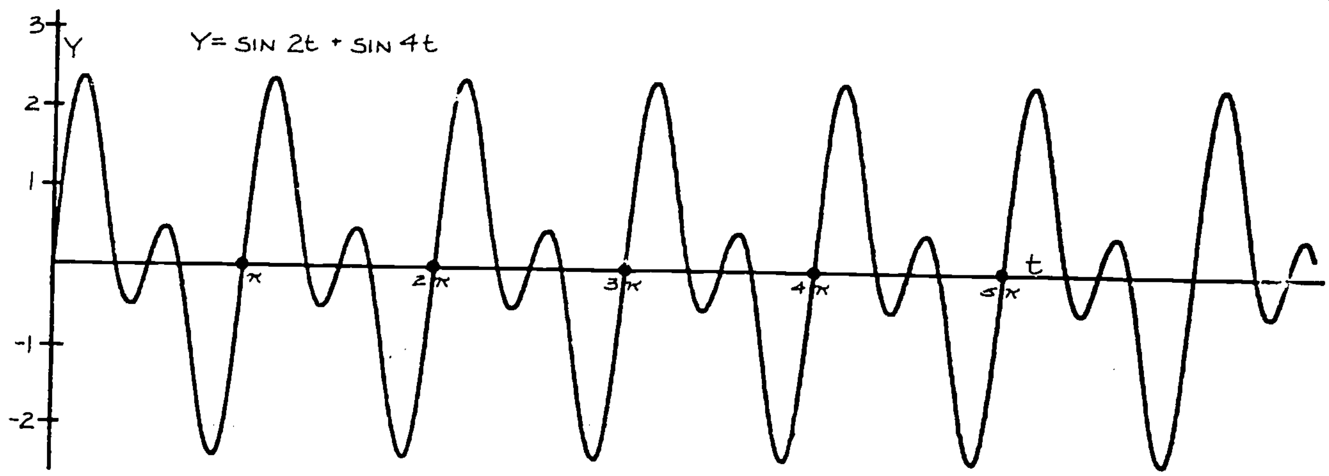
m	n	
1	4	wave and third harmonic
1	8	wave and seventh harmonic

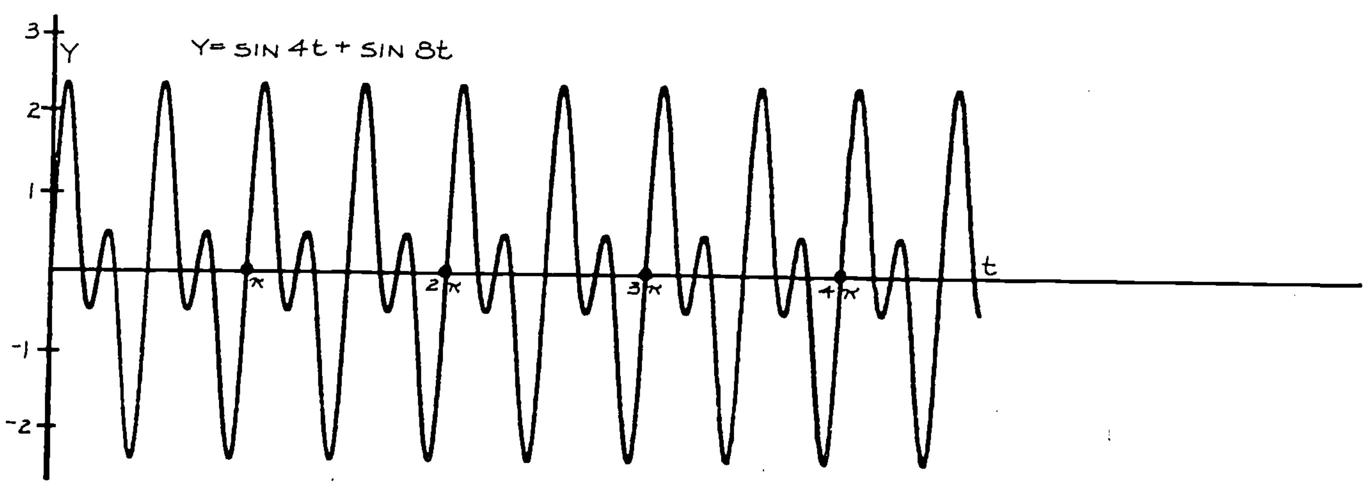
3. Have the students add the curves by means of the compass method described earlier.

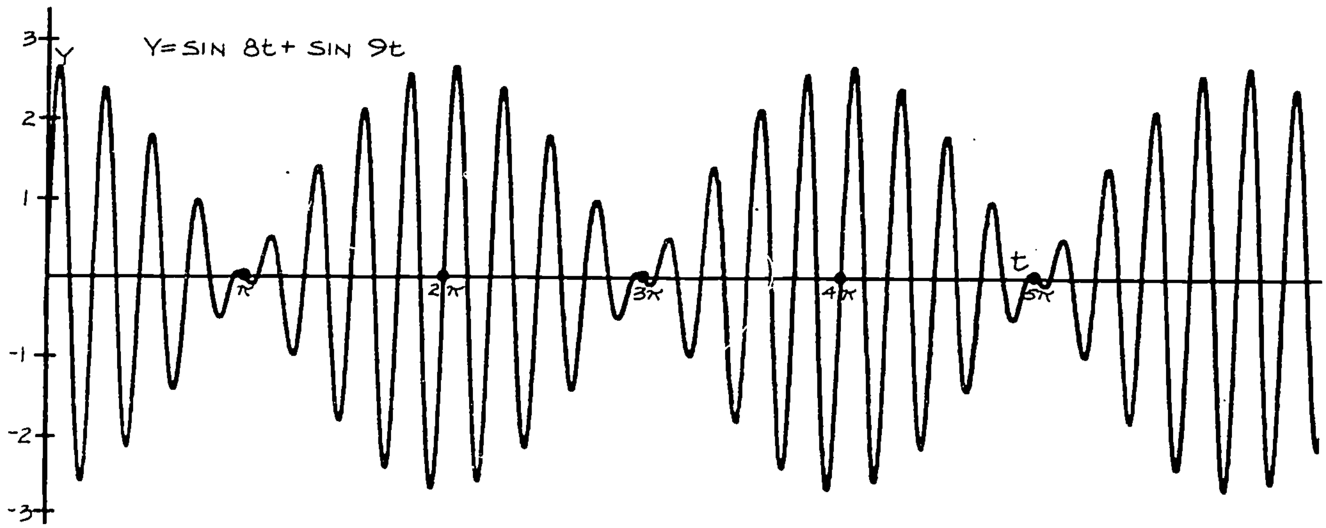
C. Explain that the graphs produced will be used to explain the behavior of an oscilloscope display of a tape recording.

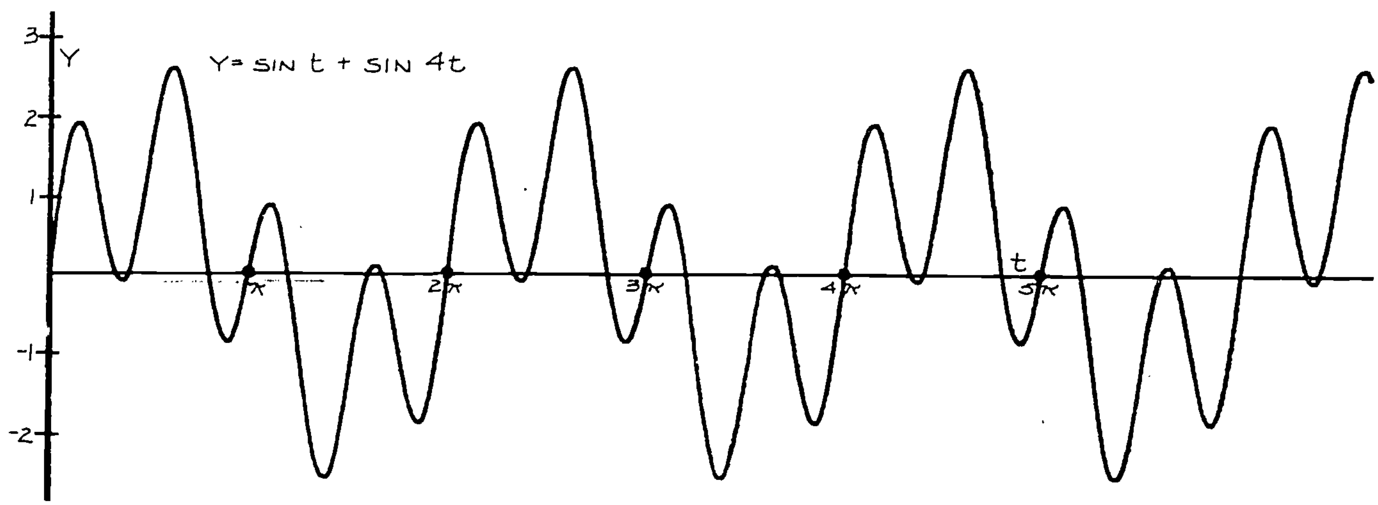
KEY--PROBLEM SET 27:

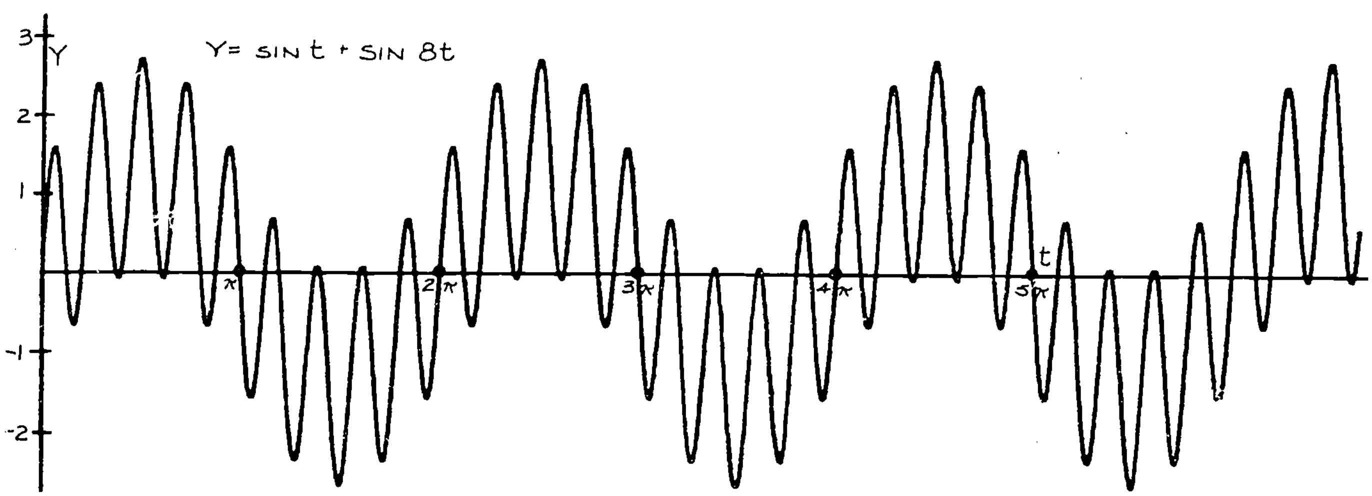







Y=SIN 8t+ SIN 9t
Y
t
3
2
1
-1
-2
-3
π
2π
3π
4π
5π


Y= SIN t + SIN 4t
Y
t
3
2
1
-1
-2
π
2π
3π
4π
5π


Y= SIN t + SIN 8t
Y
t
3
2
1
-1
-2
π
2π
3π
4π
5π

LESSON 28: BEATS AND HARMONICS

OBJECTIVES:

The student will:

- identify beat, phase, and harmonic wave relationships by means of an audiovisual demonstration of tone mixing.
- (optional) combine given frequencies to generate oscilloscope displays of the resulting waves.

PERIODS RECOMMENDED: One or two.

SUPPLIES:

Transparency Masters IV-M-28a and b

Oscilloscope

Either:
- Demonstration tape recording (available from BICP)
- Tape recorder
- Male phono jack with a patch cord and two alligator clips

Or:
- 2 BIP's
- 1 speaker
- 2 470-ohm resistors (yellow, violet, brown, and silver or gold)
- 1 11,000-ohm resistor (brown, brown, orange, and silver or gold)
- 1 200,000-ohm resistor (red, black, yellow, and silver or gold)
- 1 4.7-μF (microfarad), 10 V (volt), capacitor
- 250 cm of 24-gauge wire

4 alligator clips

wire cutter-strippers

2 ECG electrodes with attached wires and plugs

2 tuning forks, same frequency (optional)

1 rubber band (optional)

TEACHER ACTIVITIES

A. Review the graphs of sin nx + sin mx asked for in the previous lesson. Include these general ideas in your discussions.

1. All of the graphs of the form y = sin kx + sin 2kx will repeat the pattern shown in Figure 1 of Transparency Master IV-M-28a.

2. The graphs of y = sin 8t + sin 9t should look like Figure 2 of Transparency Master IV-M-28a. Discuss the similarities of the two graphs. The absolute amplitude is nearly 2 on the extreme right and left of the two graphs. In the middle the amplitude is nearly 0. This sort of pattern exemplifies beats. It is a regular, periodic variation in amplitude that occurs as a result of adding two waves with different frequencies.

B. Try to get the class to generalize from the two given graphs the appearance of the graph of sin 100t + sin 101t. The trends from Figure 1 to Figure 2 will be extended.

1. The amplitude of the sum will be a maximum on the extreme right and extreme left of its graph.

2. The amplitude of the sum will be a minimum near the middle of its graph.

3. The number of cycles in the interval $0 \le t \le 2\pi$ will increase. There are two positive peaks in the graph of $y = \sin t + \sin 2t$. There are nine positive peaks in the graph of $y = \sin 8t + \sin 9t$. There will be 101 wave crests in the graph of $y = \sin 100t + \sin 101t$.

4. The difference in amplitude between successive wave crests will decrease.

Tape-Recorder Option: Play the demonstration tape and observe the oscilloscope display. (Is there a ringing in your ears? There ought to be. Be thankful that you didn't have to make these subtle torture devices.)

The order of the recorded effects is (1) fast beats, (2) slow beats, (3) frequency f, (4) frequency 2f, (5) the sum of f and 2f $\sin \frac{f}{2\pi}t + \sin \frac{2f}{2\pi}t$.

Consult the "Oscilloscope Demonstration Instructions" which follow this section for details on setting up the demonstration.

Invite the class to describe their observations of the oscilloscope displays. They should use the words "phase," "beats" and "harmonics."

Two-BIP Option: Consult the "Oscilloscope Demonstration Instructions" which follows this section. In this option you will need skill to move from one effect to another. It is imperative that you put in some practice time on this before class if you are to accomplish it smoothly. You should demonstrate beats, at least the case $\sin kt + \sin 2kt$; with this option the possibilities are virtually endless. For example, you can mix two very dissonant tones, tune in an oscilloscope image of the beat wave, count the peaks and determine the ratio of the frequencies. More about this later.

For either option, the display of $\sin kt + \sin 2kt$ will be unstable. This is because the BIP oscillators are not perfectly stable. A variation of only one cycle every four or five seconds will produce a gradual phase shift. Therefore, the pattern shown in Transparency Master IV-M-28a, Figure 1, is only a transitional one. The graphs of $y = \sin t + \sin 2(t - \frac{\pi}{4})$ and $y = \sin t + \sin 2(t + \frac{\pi}{4})$ are shown in Transparency Master IV-M-28b. The display will be seen to gradually shift between the three phase relationships.

While observing the display of the beats, point out that the scope displays the sound wave a cycle or so at a time. The result is a sequence of oscilloscope patterns which resemble the sequence of phase-shift graphs produced in Lesson 27. When the difference in frequency between two notes is small, then beats may be likened to a gradual shift in the phase relationship between two added waves. Another way to explain beats is to look at the graph of $y = \sin 8t + \sin 9t$. The amplitude of each succeeding cycle differs little from adjacent cycles. If it were the graph of $y = \sin 100t + \sin 101t$, then the difference in amplitude between succeeding cycles would be smaller yet. Therefore, since the oscilloscope displays the sum one cycle at a time, it will appear that the amplitude of the sum is gradually decreasing.

Make these points about beats. The beat frequency that results from mixing frequencies is equal to the difference in frequency of the two added frequencies. For example, if t is in seconds and

$$f = \frac{b}{2\pi} \text{ for } y = \sin bt,$$

$$f_1 \text{ of } \sin 8t \text{ is } \frac{8}{2\pi}\frac{\text{cycles}}{\text{sec}}$$

$$f_2 \text{ of } \sin 9t \text{ is } \frac{9}{2\pi}\frac{\text{cycles}}{\text{sec}}$$

The difference in frequency is $\frac{1}{2\pi}\frac{\text{cycle}}{\text{sec}}$ or 1 cycle in 2π seconds. This may be

confirmed by referring to the graph of $y = \sin 8t + \sin 9t$. The large amplitudes on the right and left extremes correspond to maximum loudness. The small amplitude in the middle corresponds to minimum loudness. Therefore, the sum of the two notes starts out loud at $t = 0$ sec, is quietest around $t = 3.14$ sec and is loudest again at $t \approx 6.28$ sec. This is one beat cycle, the variation from maximum loudness, to quiet and back to loud. This regular variation in loudness is characteristic of beats.

Consider another example. Suppose

$$y = \sin 200t + \sin 201t$$

From our previous discussion we know that there will be 201 wave crests in the interval $0 \le t \le 2\pi$ sec. We also know that the difference in amplitude between succeeding peaks will be very small. This could be shown by graphing the function, but anyone who has had the experience of graphing $\sin 8t + \sin 9t$ will be loath to attempt this tedious task. A mathematical analysis based on the extrapolation of trends is much simpler, if it can be understood. From the previous discussion of the graphs of

$$y = \sin t + \sin 2t$$

$$y = \sin 8t + \sin 9t$$

$$y = \sin 100t + \sin 101t$$

we know that the graph of

$$y = \sin 200t + \sin 201t$$

will have maximum amplitude on the right and left extremes. Its amplitude will be a minimum in the center. The import of this is that its beat frequency will be the same as all the others, or 1 beat per 2π (≈ 6.28) seconds.

$$f_{beat} = \frac{1}{2\pi} \frac{\text{beat}}{\text{sec}}$$

$$\approx .16 \frac{\text{beat}}{\text{sec}}$$

Proceed to calculate the frequencies of the two added waves.

Again, $f = \frac{b}{2\pi}$ for $y = \sin bt$. Therefore,

$$f_1 = \frac{200}{2\pi} \frac{\text{cycles}}{\text{sec}}$$

$$f_1 \approx 31.83 \text{ cps}$$

$$f_2 = \frac{201}{2\pi} \text{ cps}$$

$$f_2 \approx 31.99 \text{ cps}$$

$$f_2 - f_1 \approx .16 \text{ cps}$$

Point out that the frequencies of these notes are on the low end of the audible range. If we were to mix the two notes, we would hear a beat about every 6.28 seconds. Once again the beat frequency is equal to the difference between the frequency of the two added waves.

Make the following generalization: "The beat frequency resulting from the addition of two notes is equal to the absolute value of the difference in their frequencies." We have not proved this, because the proof is beyond the training of the students. Provide some drill in the use of the generalization by having the class give the beat frequency of pairs of frequencies. For example, the beat frequency of 60 cps and 62 cps is 2 cps.

(Optional) Beats may be produced from two tuning forks. The two forks should be the same frequency. The frequency of one may be altered slightly by wrapping a rubber band around one of the tines. The beat frequency may be varied by moving the rubber band up and down. Demonstrate how to obtain beats from the pair. The two forks may be passed around while the class is working on the problem set.

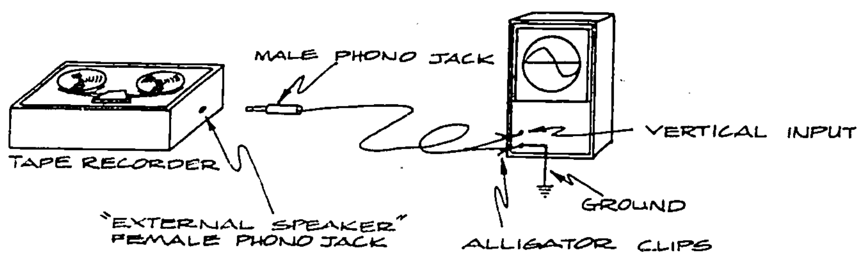
OSCILLOSCOPE DEMONSTRATION INSTRUCTIONS

There are two ways to go about this demonstration. The images of beats and harmonics may be displayed by playing the demonstration tape on a tape recorder. Or, two BIP's may be used to generate the images. The tape-recorder option has the advantage of convenience while the two-BIP option has the advantage of versatility. Ideally a combination of the two would be the best. The tape recording could be used in the initial phase to make instructional points with a minimum of extraneous noises and knob twiddling. Later the two BIP's can be wired up to provide the opportunity to mix any two waves.

Of course, the ideal arrangement would be to have as many BIP-oscilloscope stations as possible to give all the students an opportunity to mix waves. This option would require at least one more period, but it would be worth it.

TAPE-RECORDER OPTION:

Connect the tape recorder to the scope as shown in the diagram below.



Details:

A. The alligator clip that is connected to the shielding of the cable should be connected to the scope's ground. The shielding is a braided metal sheath. The "hot" wire is located in the center of the braided sheath.

B. On the particular tape recorders we have tested we were able to view the oscilloscope display and hear the sound simultaneously. This was achieved by not pushing the phono jack in all the way. We pushed it in only until contact was first made. Probably you will be able to do the same thing with your recorder; however, there is no guarantee that your recorder will provide this option.

C. It will require some patience to find the proper balance between the volume control of the recorder and the vertical gain control of the scope. Adjust these two controls until you obtain the sharpest possible picture.

D. Set the recorder to the $3\frac{3}{4}$ ips speed during the "tuning in" phase.

E. The order of recorded effects is

1. fast beats
2. slow beats
3. sin t
4. sin 2t
5. sin t + sin 2t

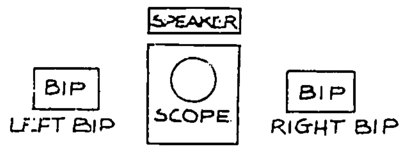
Item 3 above occupies a long stretch of tape. This is for the purpose of tuning in. Use this stretch of tape to adjust the period and amplitude of the display. The amplitude of this note is supposed to be "one" in relation to the rest of the recorded notes. It isn't quite. The tape is not perfect. Adjust the equipment until it is possible to play the entire tape and contain all of the displays on the screen.

F. When you are finished with your knob adjustments, set the recorder on the fastest speed, i.e., $7\frac{1}{2}$ ips, and readjust your sweep frequency accordingly.

G. All adjustments should be completed before class commences.

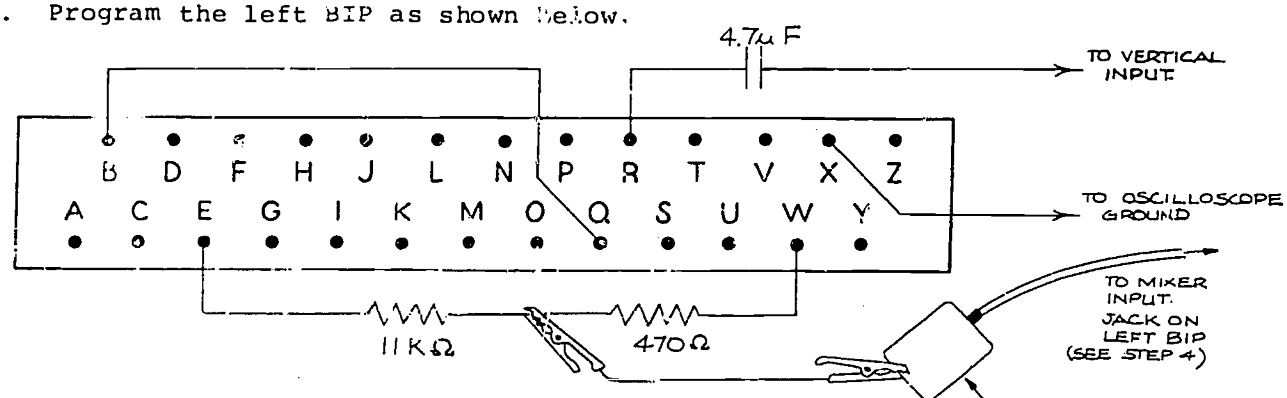
TWO-BIP OPTION:

A. Arrange the two BIP's, oscilloscope and speaker as shown below. This may seem trivial now, but it won't later. Each BIP is programmed differently and functions differently.



FRONT VIEW

B. Program the left BIP as shown below.



Left BIP Programming (Back View)

Explanation of Symbols:

11K Ω — An 11,000-ohm resistor. It will have four bands in the order brown, brown, orange and either silver or gold. The first band is closest to one end.

470 Ω — A 470-ohm resistor, color code yellow, violet, brown, and either silver or gold.

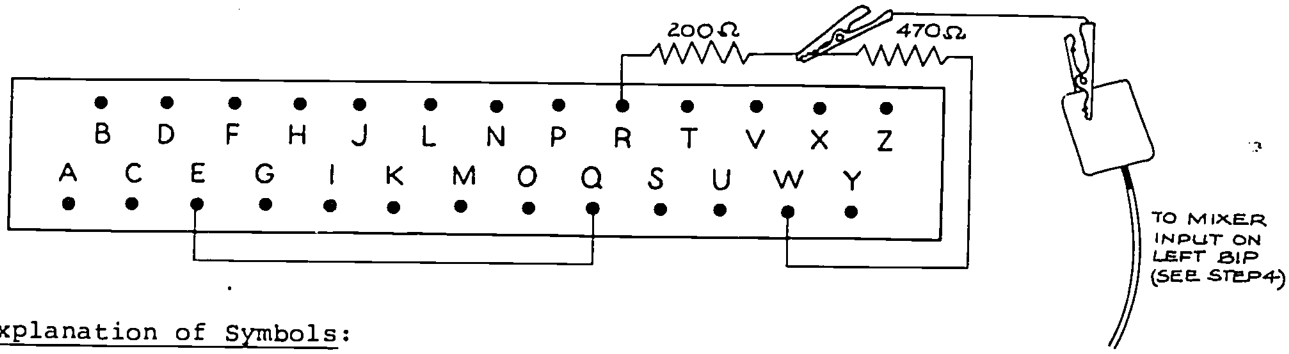
 An alligator clip with a long wire attached.

4.7 µF — 4.7-microfarad capacitor.

CAUTION: Do not plug the resistor or capacitor leads directly into the programming terminals on the BIP. The resistor wires are larger than the terminals are designed to accept, although they may be forced in. However, if they are forced in, the terminal will be sprung and will no longer make good contact with the size wire for which it was designed. Sprung terminals must be replaced.

To connect a resistor or capacitor to a terminal, twist or solder a wire of the proper size to the resistor lead before making connection.

C. Program the right BIP as shown below.

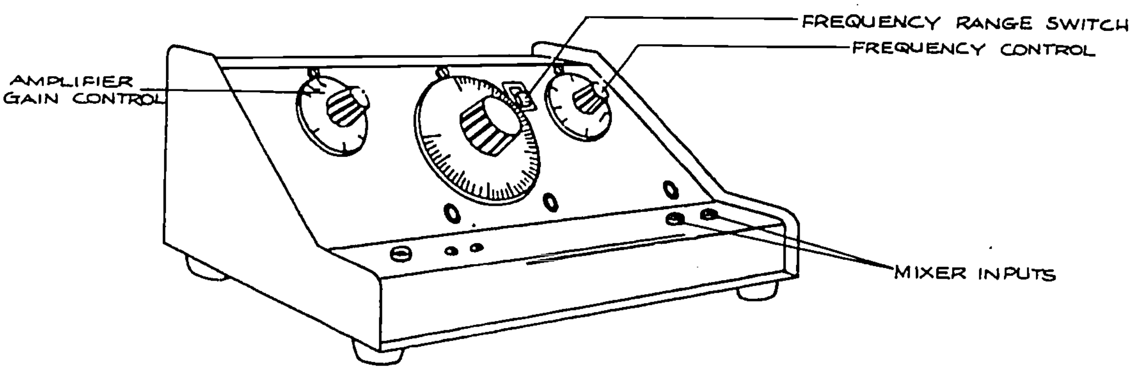


Explanation of Symbols:

200K Ω } A 200,000-ohm resistor, color code red, black, green, yellow and silver or gold.

470 Ω } A 470-Ω resistor, color code yellow, violet, brown and silver or gold.

D. The audio-frequency outputs of each BIP are mixed in the left BIP. The wires connected to the alligator clips feed the audio outputs into the mixer inputs which are found at the lower right of each BIP's front panel (see below).



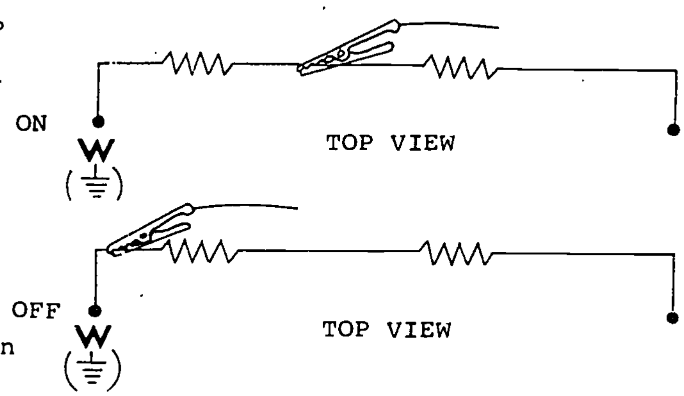
Use another pair of alligator clips to connect the alligator clip wires to the ECG electrodes. Then plug the electrode wires into each of the female phono jacks on the left BIP.

E. Connect the speaker across the oscilloscope ground and vertical input.

F. Plug in the BIP's, turn on the oscilloscope, and you are ready to begin mixing.

G. To get the output of a single BIP displayed, it is necessary to change the position of the alligator clips on the resistors behind the BIP's. At right are two diagrams which describe the "on" and "off" positions of the clips.

The views are from the perspective of leaning over the front of the BIP's, looking down. Terminal W is a ground connection. When the alligator clip is on the W side of the resistors, that particular BIP is "off." When the clip is between the resistors, the BIP is "on."



H. Turn the right BIP off and tune in the oscilloscope to the output of the left BIP. Adjust the oscilloscope so that the amplitude of the displayed sine curve is one.

Do not use the "sync" feature of the scope when you are adjusting the sweep frequency. You may switch it on if you wish when adjusting the amplitude (vertical gain on the scope). The reason for this will be explained a little later.

I. Adjust the oscilloscope's horizontal gain so that one cycle falls as precisely on the grid divisions as possible. It is best to use the distance between two adjacent maxima or minima for this rather than the standard sine cycle.

Later one, when you adjust the frequency of the right BIP, it will be very important to know the horizontal distance on the oscilloscope screen that corresponds to one cycle for the left BIP. You may even want to use a grease pencil to flag the divisions.

J. Now turn the left BIP off (by moving the alligator clip) and the right BIP on. The amplifier gain (db) control on the right BIP adjusts the amplitude of the output of the right BIP only, whereas the gain control of the left BIP controls the amplitude of the combined signal.

Use the gain control of the right BIP to adjust the amplitude of the displayed curve to one.

K. Use the frequency control of the right BIP to obtain the desired frequency. Be sure the "sync" feature of the oscilloscope is not on.

If you want to display beats, approximate closely the frequency of the left BIP.

If you want to display sin t + sin 2t, adjust the frequency so that two cycles appear in the same horizontal distance that one was shown for the left BIP. After the frequency is adjusted, the amplitude may need readjustment, however, make all adjustments with the BIP controls. Do not touch the scope controls except for the horizontal and vertical position controls. The vertical gain and sweep frequency should remain just as they were when you finished tuning in the left BIP.

L. Now you are ready to mix signals. Turn the left BIP on and switch the oscilloscope to sync. You should get the desired waveform.

KEY--PROBLEM SET 28:

1. 3 cps
2. .34 cps
3. $.0003 \times 10^4$ cps or 3 cps
4. $.01 \frac{\text{kilocycles}}{\text{sec}}$
5. .025 cps
6. 7 cps
7. 2 cps
8. a. II
   b. III
   c. I
9. a. I
   b. IV
   c. 3
   d. I, III, V, II, IV
   e. IV
   f. 1 beat per 2π time units or $\frac{1}{2\pi}$ cycles per unit time.

LESSON 29: REVIEW

OBJECTIVE:

The student will solve problems relating to the objectives of Lessons 23 through 29.

PERIODS RECOMMENDED: One.

KEY--REVIEW PROBLEM SET 29:

1. $a = 3$

2. $a = 20$

3. $a = 47$

4. $a = 10^3$

5. $f = \frac{1}{40} \frac{\text{cycle}}{\text{sec}}$

   $T = 40 \frac{\text{sec}}{\text{cycle}}$

6. $f = \frac{1}{24} \frac{\text{cycle}}{\mu\text{sec}}$

   $T = 24 \frac{\mu\text{sec}}{\text{cycle}}$

7. $f = \frac{b}{2\pi} \frac{\text{cycles}}{\text{year}}$

   $T = \frac{2\pi}{b} \frac{\text{years}}{\text{cycle}}$

8. $f = 1 \frac{\text{cycle}}{\text{min}}$

   $T = 1 \frac{\text{min}}{\text{cycle}}$

9. $f = \frac{113}{2\pi} \frac{\text{cycles}}{\text{millisecond}}$

   $T = \frac{2\pi}{113} \frac{\text{millisecond}}{\text{cycle}}$

10. $\lambda = 6\pi \frac{\text{meters}}{\text{cycle}}$

11. $\lambda = 2 \frac{\text{km}}{\text{cycle}}$

12. $\lambda = \frac{2\pi}{c} \frac{\text{meters}}{\text{cycle}}$

13. $\lambda = \frac{2\pi}{33} \frac{\text{km}}{\text{cycle}}$

14. $\lambda = \frac{1}{7} \frac{\text{mm}}{\text{cycle}}$

15. a. V

    b. I

    c. V, III, II, IV, I

16. a. 1 cps

    b. 3 cps

    c. 3 cps

17. d and e

60

LESSON X1: ABSOLUTE VALUE AND SUMMATION NOTATION

OBJECTIVES:

The student will:

- find the absolute value of an expression.
- evaluate sums written in summation notation.

PERIODS RECOMMENDED: One

INTRODUCTION:

The following eight Mathematics Lessons, which treat the subject of vision, have been divided into three "floating" groups. Each group depends on data from a Science activity. The first group treats data from "The Index of Refraction" (LA-28). The second group depends on data from "Determining the Focal Length of Lenses" (LA-30). The vision sequence ends with an analysis of the data obtained from the "Vision Screening Field Trip" (Activity 35).

The Mathematics lesson groups are listed below along with the scheduling relative to the science class.

To be started the same day as Science Lesson 28:

X1: ABSOLUTE VALUE AND SUMMATION NOTATION

X2: SNELL'S LAW

X3: THE MEAN AND THE MEAN DEVIATION

X4: STANDARD DEVIATION AND WEIGHTED MEAN

To be started the same day as Science Lesson 31, or as soon as possible thereafter:

Y1: CALCULATION OF FOCAL LENGTH

Y2: STANDARD DEVIATION AND FOCAL LENGTH

Y3: REVIEW

To be started immediately after Science Lesson 35:

Z: EVALUATING THE VISION-SCREENING DATA

You should follow the suggested scheduling of lessons as closely as possible. If you are a few days ahead of this schedule we suggest that you devote the extra time to drill in those areas where your class is weak or move ahead into Unit V.

A description of the Science experiments has been omitted from the Mathematics materials. Therefore, you will need to refer to the Science laboratory manual in order to become familiar with the relevant activities. We will cite the related activities under the heading "REFERENCE."

Finally, it should be obvious that reading the related activities alone is only minimal preparation for teaching the Mathematics sequence. The ideal preparation would involve being present while the class performs the experiments. Underlying all this, there is no substitute for a good, solid communications relationship with the Science instructor. You should be able to rely heavily on the Science instructor to describe each of the activities and to answer any questions you might have.

The vision sequence in Science is organized to provide the groundwork for a culminating activity, the administration of the Snellen test, which occurs in Lesson 35. In this activity the class will visit an elementary school and test the visual acuity of students there. From the Mathematics point of view the

preparation for the Snellen test falls into two general subject areas. The first is refraction and lenses. The second is statistics, which can help students make sense of the data obtained in the vision-screening activity. Today's lesson is a review of absolute value and summation notation intended to prepare students to work with statistical formulas.

At several points in the vision lessons you will find references to transparency masters with sample calculations. These masters are located at the end of this book and can be used either to make transparencies or as a guide for developing examples on the board. You may want to scan the masters now to decide whether the type size will project clearly in your particular classroom setting.

Please note in the SUPPLIES section of Lesson X2 that you will need two of Data Sheet X2 for each student. You will need to make copies of this data sheet before class.

KEY--PROBLEM SET X1:

1. 8	6. 7	11. 12	14. a. 36
2. 6	7. 26	12. 4	b. 18
3. 0	8. 5	13. a. 6	c. 36
4. 1.34	9. 2	b. 6	
5. 6	10. 5	c. 6	

LESSON X2: SNELL'S LAW

OBJECTIVE:

The student will calculate the index of refraction of air, water and glycerine from data collected in the Index of Refraction Science Laboratory Activity.

PERIODS RECOMMENDED: One

REFERENCE:

Laboratory Activity 28: The Index of Refraction

SUPPLIES:

Math Data Sheet X2 (two per student)

Graphs from LA-28 (students' own--three per pair of students; obtain from Science instructor)

Protractors (one per student)

TEACHER ACTIVITIES:

A. Return to the students the graphs that you have received from the Science instructor. Distribute two copies of Math Data Sheet X2 to each student, along with protractors. It will be necessary to refer to the Table of Trigonometric Functions at the end of the Student Text.

The task to be performed is the measurement of the angles α and β on the student graphs and the calculation of the index of refraction for each pair of angles. The procedure is thoroughly explained in Section X2 of the Student Text.

Before they begin, conduct a brief discussion of the "air-to-air" data. With reasonable care in experimental procedure, the lines forming α and β should be straight, and α should equal β. Discuss the value of an experimental control.

MATH DATA SHEET X2

Name ______________________

Air to ____________

Trial #1	α	β	$\sin\alpha$	$\sin\beta$	n	$\lvert n_i - \bar{n}\rvert$	$(n_i - \bar{n})^2$
				$\sum$ (sum)			
					Mean	Mean Deviation	Standard Deviation

$$n = \frac{\sin\alpha}{\sin\beta}$$

There is a blank at the head of the data table. Explain that "water" and "glycerine" are to be entered respectively on the two sheets. Caution the students to get the appropriate data on each sheet (i.e., to get the "water" data on the "water" sheet, etc.).

Have the students measure and record the 16 angles, find their sines and calculate the eight values of n. Explain that the remainder of the tables will be completed in subsequent lessons.

Collect the data sheets and graphs. Make certain that all of the data sheets are signed. They will be used in the next lesson.

KEY--PROBLEM SET X2:

1. glass
2. 75 grams sucrose per 100 g solution
3. $n_{\text{carbon disulfide}} \approx 1.63$
4. $c_{\text{quartz}} \approx 1.95 \times 10^8 \frac{\text{m}}{\text{sec}}$
5. a. angle of incidence = α

   angle of refraction = β

   b. $\alpha \approx 47°$

   c. $c_{\text{water}} \approx 2.19 \times 10^8 \frac{\text{m}}{\text{sec}}$

LESSON X3: THE MEAN AND THE MEAN DEVIATION

OBJECTIVE:

The student will:

·calculate the mean of the index of refraction measurements for water and for glycerine.

·calculate the corresponding mean deviation for each set of data.

PERIODS RECOMMENDED: One

SUPPLIES:

Math Data Sheet X2 (from Lesson X1--two for each student)

TRANSPARENCIES:

Transparency Masters IV-M-X3a, b

OVERVIEW AND REMARKS:

This lesson begins the statistical treatment of the optics data obtained thus far. In this and the following lesson, the class will be introduced to four basic statistics--the mean, the mean deviation, the standard deviation, and the weighted mean. The relationships between these statistics are quite complicated. For the purposes of these lessons it will suffice if the students simply know how to use the appropriate formulas.

TEACHER ACTIVITIES:

A. Return the data sheets indicated above to their owners. Point out that the object of this lesson and the next will be to answer two questions about the experimental data that have been collected.

1. When repetition of an experiment produces several values for the index of refraction (n), how can a single value of n be selected which will best represent the set of individual values?

2. How can we estimate the degree of randomness in the set of values for n?

Relate these questions to the data sheets. Point out that the students have four numbers for the refractive index of water and four for the refractive index of glycerine. The first question relates to each of these sets of data.

Discuss the fact that the mean of a set of values will, in general, differ less from the individual values than the latter will differ from one another. The arithmetic mean is the same as the "average" from arithmetic. The mean is usually shown as $\bar{X}$, and read as "ecks bar."

Transparency IV-M-X3a can be used to present a sample calculation of the mean. When the students understand the procedure, instruct them to find the mean of each of the two sets of refractive-index measurements.

B. Initiate a discussion of the second question. Error is associated with the measurement of angles in the refraction experiments. Suppose both angles were measured a little high for one pair of angles, a little low for another pair, one high and one low for another pair, etc. As a result of these chance errors the n_i will be spread out. The spread is caused by random measurement errors. It is important to be able to measure in some way the degree of scattering because it helps us judge the reliability of our results. Two such measures will be considered: The mean deviation and the standard deviation. The mean deviation will be discussed today.

Transparency IV-M-X3b can be used to present a sample calculation of the mean deviation using the same data as in Transparency IV-M-X3a. After discussing the calculation, instruct the students to find the mean deviation for each of their sets of data. When they have completed the calculations, collect the data sheets. These will be returned to them in Lesson X4, when standard deviation will be discussed.

C. Section X3-3 of the text may need some careful explanation. The section is intended to convey the idea that errors often cancel each other to some extent. Therefore the uncertainty, which we discussed last year, represents the largest possible error. In general, errors do not often follow such a pattern.

KEY--PROBLEM SET X3:

1. a. $\bar{x} = 2$

   b. $\frac{2}{3}$

2. $\bar{x} = 3$, mean deviation = $\frac{6}{5}$

3. $\bar{x} = 4$, mean deviation = $\frac{12}{7}$

4. $\bar{x} = 1.39$, mean deviation = .015

5. $\bar{x} = 1.40$, mean deviation = .015

6. They are the same.

LESSON X4: STANDARD DEVIATION AND WEIGHTED MEAN

OBJECTIVES:

The student will:

•calculate the standard deviation for the set of refractive index data for water.

•calculate the weighted mean of two different data sets for water.

PERIODS RECOMMENDED: One

SUPPLIES:

Math Data Sheet X2 (from Lesson X3)

Calculators (optional)

TRANSPARENCIES:

Transparency Masters IV-M-X4a, b

TEACHER ACTIVITIES:

A. Return the materials collected at the end of the last period. Explain that this lesson will be concerned with answering two questions. These are:

1. How can we estimate the degree of randomness in the set of values for n?

2. How might a value of n for water be determined that uses information from several student data sets?

The mean deviation introduced in the last lesson presents itself as one answer to Question 1. However, the mean deviation is not a particularly useful statistic for the purposes of this lesson sequence. Generally speaking, this is because the mean deviation is more naturally associated with the median than the mean. The techniques required to show this are too advanced for this mathematics course.

B. The standard deviation represents the measure of variation sought as an answer to Question 1. Transparency IV-M-X4a can be used to present a sample calculation of the standard deviation. Instruct the students to calculate the standard deviation for their water refractive-index data. The square root table at the end of the Student Text will be of help in the calculations. Each standard deviation should be expressed to the nearest thousandth.

C. Section X4-2 of the text discusses the relationship between standard deviation and frequency distributions. In the chi-square lessons the students worked with several histograms having the same general shape as a continuous normal distribution. We have not stressed the process of arriving at a continuous distribution, but you might explain that such a distribution is obtained when very large data sets are collected. One interesting example of a normal distribution can be found on the steps of a very old building. The steps are worn down most in the spot where the greatest number of people place their weight. They are worn down to a lesser degree as one moves away from that spot. The result is a "footstep" frequency distribution.

For large data sets the standard deviation is a good indicator of how the data are distributed. About 68% of measurements lie within one standard deviation of the mean, 95% within two standard deviations, and so on. This relationship is used in converting achievement test scores into percentiles.

D. Returning to the vision data, take up a discussion of the second question. Each student data set for water has an associated mean refractive index and standard deviation. The results from several data sets can be combined using the weighted mean. The formula for the weighted mean is

$$\bar{x}_w = \frac{\sum_{i=1}^{n} w_i \bar{x}_i}{\sum_{i=1}^{n} w_i}, \quad w_i = \frac{1}{s_i}$$

$\bar{x}_w$ = weighted mean

$\bar{x}_i$ = mean for data set i

s_i = standard deviation for data set i

w_i = weight for data set i

Point out that the smaller the standard deviation (s_i), the larger the weight (w_i) becomes. Hence the formula for the weighted mean gives more credit (weight)

to experiments with less variation. Since variation indicates lack of precision in the experiment, this is as it should be.

Divide the class so that each student will be computing the weighted mean for two different water data sets. Transparency IV-M-X4b can be used to present a sample calculation of the weighted mean.

It might be worthwhile to demonstrate how to compute a weighted mean for all the data sets in the class. If you have a calculator you can actually carry out the necessary calculations.

The computation of the weighted mean for the glycerine data is optional. Given the complexity of the calculations, there may not be sufficient time. When all calculations are completed, compare class values for the refractive index to the following "book" values expressed to the nearest thousandth.

water: 1.333

glycerine 1.474

This lesson completes the analysis of the refractive-index data. The data sheets will not be needed in future lessons.

LESSON Y1: CALCULATION OF FOCAL LENGTH

OBJECTIVES:

The student will:

- calculate focal length from measurements of image distance and object distance.

- calculate the mean focal length for three different lenses.

PERIODS RECOMMENDED: One

REFERENCE:

Laboratory Activity 30: Determining the Focal Length of Lenses

SUPPLIES:

Science Data Sheet 30 (2 per pair of students)

TRANSPARENCIES:

Transparency Masters IV-M-Y1a, b

OVERVIEW AND REMARKS:

This is the first of two lessons on lenses and focal length. Lesson Y3 is a review of Lessons X1-Y2. In this lesson, data from Laboratory Activity 30 are used to calculate the focal lengths of several lenses. In the next lesson, the students work with formulas that can be used to predict the focal length of a lens. The statistical tools developed in the lessons on index of refraction are applied in this sequence to the lens data.

This lesson should be presented the same day as Science Lesson 31, or as soon after as possible.

TEACHER ACTIVITIES:

A. Distribute the data sheets from the lens experiment which have been given to you by the Science instructor. Each pair of lab partners did two experiments,

NAME: ______________________

SCIENCE DATA SHEET 30

Watch glass depth: d = ________

Watch glass width: w = ________

$r = \frac{w}{2}$ = ________

R = ________

Lab partner #1 ________________

Lab partner #2 ________________

	0% Sucrose			25% Sucrose			50% Sucrose		
Screen position (cm)									
d_o: object distance (cm)									
d_i: image distance (cm)									
Experimental focal length (cm)	f_1	f_2	f_3	f_1	f_2	f_3	f_1	f_2	f_3
Mean focal length, $\bar{f}$ (cm)									
Deviations from mean focal length: $f_i - \bar{f}$ (cm)									
$(f_i - \bar{f})^2$ (cm^2)									
Standard Deviation, s (cm)									
Theoretical focal length, f_θ (cm)									

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one with a large lens and one with a small lens. One sheet should go to each lab partner.

Explain to the class that each student will be responsible for nine focal-length calculations and three mean focal lengths by the end of the class period. The remainder of the calculations (i.e., standard deviation and theoretical focal length) will be done in class during the next class period. Any remaining time can be spent by getting a start on the problem set.

B. Transparency IV-M-Y1a can be used to review the idea of focal length. It is the distance between the center of the lens and the point at which parallel rays of light are focused. The equation in the center of the transparency relates the focal length to the object distance, d_o, and the image distance, d_i. The students have obtained experimental values for d_o and d_i. The formula can be used to calculate the focal length. Transparency IV-M-Y1b shows a sample calculation. Notice that the formula has been solved for f. When the discussion is completed instruct the students to complete on their data sheets the row labeled "Experimental focal length (cm)."

C. Instruct the students to fill in the row labeled "Mean focal length, $\bar{f}$ (cm)." When calculations are completed collect the data sheets. They will be used again in the next lesson.

KEY--PROBLEM SET Y1:

1. a. (1) $\frac{1}{d_o} + \frac{1}{d_i}$ b. (2) $\frac{d_o \cdot f}{d_o - f}$ c. $d_i = 20$ cm

2. a. refraction b. curvature c. (3) $\frac{1}{f} = (n - 1)(\frac{2}{r_1})$ d. $f = \frac{r_1}{2(n - 1)}$

3. $r = 9$ cm

4. $n \approx 1.53$

5. a. $d_o = 80$ cm b. $d_i = 2$ cm c. $f \approx 1.95$ cm

6. a. $d_o = 18$ cm b. (2) decreases

7. $d_i \approx 4.95$ cm

LESSON Y2: STANDARD DEVIATION AND FOCAL LENGTH

OBJECTIVES:

The student will:

- calculate the standard deviation of each of the three sets of focal-length measurements that were dealt with in Lesson Y1.

- calculate the focal length of each of the three lenses from its radius of curvature and its refractive index.

PERIODS RECOMMENDED: One

SUPPLIES:

Science Data Sheet 30 (2 per pair of students)

TRANSPARENCIES:

Transparency Masters IV-M-Y2a, b

TEACHER ACTIVITIES:

A. Explain that today the students will complete the calculations for the lens experiment. Return the students' data sheets.

Review the procedure for computing the standard deviation. First the variance s^2 is computed.

$$s^2 = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n - 1}$$

In order to obtain the standard deviation, s, the square root is taken.

$$s = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n - 1}}$$

Point out that in the lens experiment n = 3 because there are three independent determinations of focal length for each lens-solution combination. The general statement of s reduces, for this experiment, to

$$s = \sqrt{\frac{\sum_{i=1}^{3} (f_i - \bar{f})^2}{2}}$$

where $\bar{f}$ is the mean focal length and f_i the independent measurements of focal length.

Instruct the students to perform the three standard deviation computations based on their focal-length data. They should refer to the Square Root Table at the end of the Student Text and express each standard deviation to the nearest hundredth. Transparency IV-M-Y2a can be used to present a sample calculation.

B. Next instruct each student to compute the radius of curvature, R, of his or her watch-glass lens. The top half of Transparency IV-M-Y2b develops the necessary formula. Be sure to point out the relevant section of Science Data Sheet 30 to be filled out.

C. The last order of business is the completion of the last row of Science Data Sheet 30. This involves the computation of the theoretical focal length, f_θ. The derivation of the relevant formula begins with the equation

$$\frac{1}{f_\theta} = (n - 1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

where n is the index of refraction of the lens material and R_1 and R_2 are the radii of curvature of the two faces of the lens. Assuming that the two watch glasses used to make the lens are roughly identical, $R_1 = R_2 = R$. Therefore,

$$\frac{1}{f_\theta} = (n - 1)\left(\frac{2}{R}\right)$$

$$f_\theta = \frac{R}{2(n - 1)}$$

This formula appears on Transparency IV-M-Y2b along with the three indexes of refraction necessary for student calculations.

LESSON Y3: REVIEW

OBJECTIVE:

The student will solve problems related to the objectives for Lessons X1-X4 and Y1-Y2.

PERIODS RECOMMENDED: One

KEY--PROBLEM SET Y3:

1. a. index of refraction
   b. angle of incidence
   c. angle of refraction
2. $2.57 \times 10^8 \frac{m}{sec}$
3. increases
4. D
5. A
6. B
7. D
8. B
9. 10
10. $\frac{4}{3}$
11. 10
12. $\frac{10}{3}$
13. B
14. 2
15. 8
16. 3.61
17. 11
18. 6.56
19. 2
20. ~ 1.54
21. a. object distance
    b. image distance
    c. $f = \frac{d_i d_o}{d_i + d_o}$
    d. $d_i = \frac{d_o \cdot f}{d_o - f}$
22. a. 20 cm
    b. 11 cm
    c. 10.1 cm
    d. closer together
    e. 10
23. 24 cm
24. a. 1.4
    b. $\sim 2.14 \times 10^8 \frac{m}{sec}$
25. 1.6

LESSON Z: EVALUATING THE VISION-SCREENING DATA

OBJECTIVE:

The student will determine the mean, median and mode of different sets of vision-screening data for the purpose of comparison.

PERIODS RECOMMENDED: Two

REFERENCES:

Description of Snellen Test, Science Text Section 33

Snellen Self-Testing, Laboratory Activity 34

Mean, Median, Mode, Science Text Section 34

Vision-Screening Field Trip, Activity 35

SUPPLIES:

Class sets of visual-acuity data from the self-testing activity of Science Lesson 34 and the primary-school screening of Lesson 35.

TRANSPARENCIES:

Transparency Master IV-M-Z

OVERVIEW AND REMARKS:

This lesson should be presented as soon as possible after Science Lesson 35 (Vision-Screening Field Trip). The purpose is to explore data relationships and compare results for various subgroups. In addition to the suggestions given below, you might also consider applying chi square to some of the data to test for non-random patterns.

TEACHER ACTIVITIES:

A. Section Z-1 of the text discusses the effect of cutoff points on the pattern of referrals. The important point to stress is that raising the standard for passing, results in a decrease of the number of false negatives (desirable) and an increase in the number of false positives (undesirable). You might want to reinforce the terminology--positives, negatives, false positives, false negatives--by discussing the following table which does not appear in the text.

Cutoff	Number of passes	Number of referrals	Number of those referred who do not need care	Number of children with undetected vision problems
20/25	70	30	14	1

Here the number of negatives is 70, the number of positives is 30, the number of false positives is 14 and the number of false negatives is 1. Of those referred, 30 - 14 or 16 actually needed care.

B. Inform the students that during this class period they will be organized into committees in order to determine the mean, median and mode of different sets of vision-screening data. Once these are determined, a discussion will be held on whether there are significant differences in visual acuity between the different sets of data. The primary-school data may also be compared with results published in the Orinda study. These results are shown on Transparency IV-M-Z. (You may wish to make up a similar chart based on your own community.)

Possible sets of data for comparison might be

1. Biomed-class male

2. Biomed-class female

3. Primary-school male

4. Primary-school female

C. Divide the class up into committees in such a way that the necessary calculations are completed.

D. Once the calculations have been made, illustrate how the combined mean of males and females in one class may be obtained.

$$\frac{(\text{\# of females})\cdot(\text{mean scale of females}) + (\text{\# of males})\cdot(\text{mean scale of males})}{(\text{\# of females}) + (\text{\# of males})}$$

E. Possible springboards for discussion might include:

1. Is there a significant difference in visual acuity between the sexes?

2. Is there a significant difference in visual acuity between the Biomed class and the primary-school class?

3. Is there a significant difference in visual acuity between the primary-school children tested and the Orinda Study group of children?

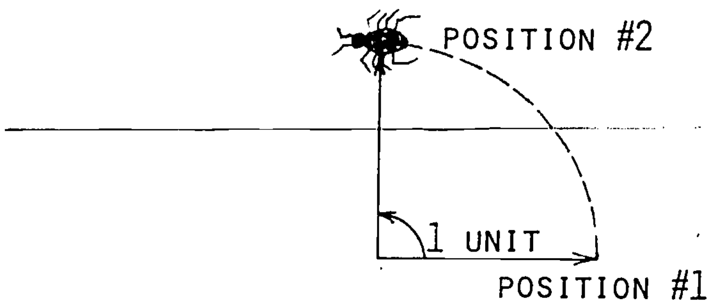
4. If it is agreed that there is a difference in #3, then a discussion of the differences in the samples might be held.

It would be useful to point out that a statistical measure such as chi square could be used to determine the significance of the possible differences suggested above.

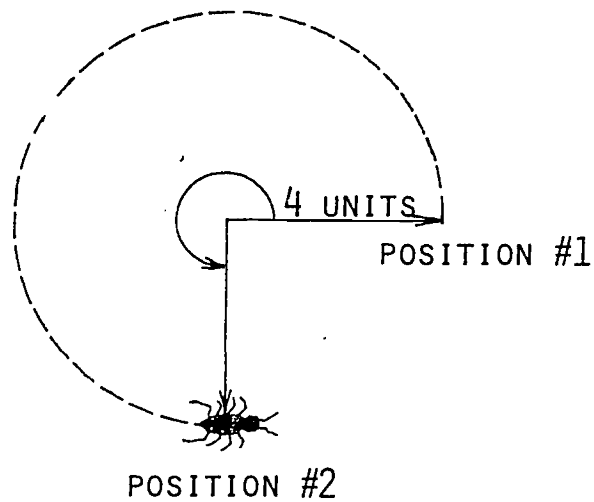
F. If the charts used in the screening did not have all of the lines indicate in the Orinda data, then the mean of the Orinda data must be recalculated if comparisons between means are to be made. For example, if the chart used did not have a 20/40 line, then the number of eyes in the 20/40 category of the Orinda Study must be rescaled at 8 instead of 7, because all of these eyes would have had a scale of 8 in the screening as performed by the Biomed class.

MEASUREMENT OF ANGLES IN DEGREES, REVOLUTIONS AND RADIANS

EXAMPLE 1



EXAMPLE 2



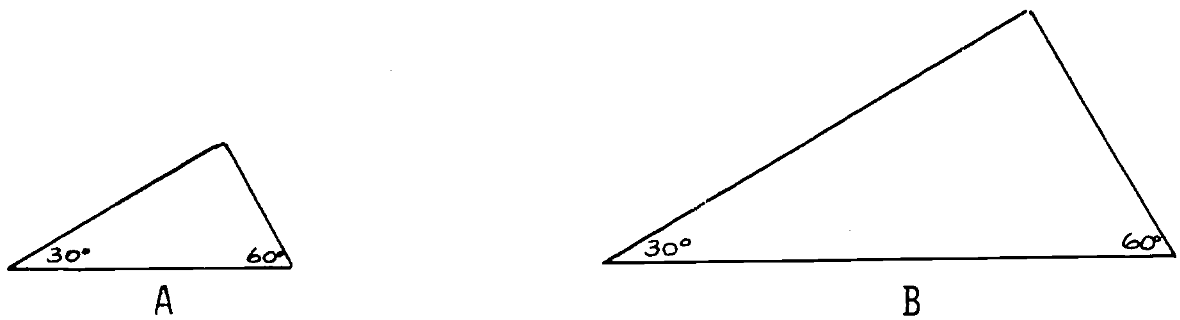
1. DEGREES: IN DEGREES, WHAT ANGLE DOES THE BUG GENERATE IN MOVING FROM POSITION #1 TO POSITION #2?

2. REVOLUTIONS: WHAT FRACTION OF THE TOTAL CIRCLE HAS THE BUG COVERED?

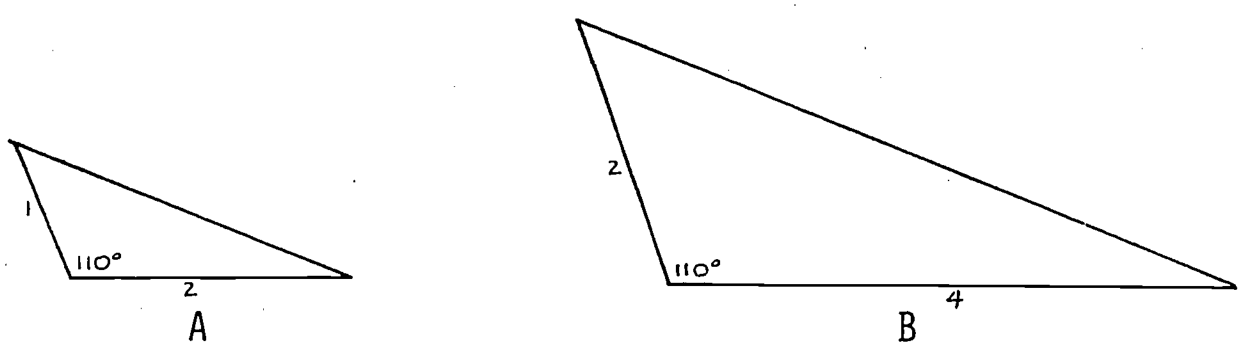
3. WHAT IS THE CIRCUMFERENCE OF THE COMPLETE CIRCLE?

4. CONSEQUENTLY, WHAT DISTANCE HAS THE BUG TRAVELED?

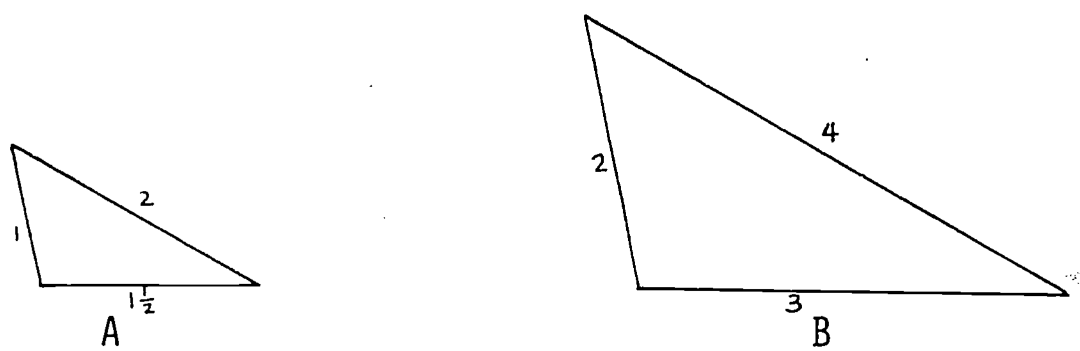
5. RADIANS: DIVIDE THE DISTANCE TRAVELED BY THE RADIUS OF THE CIRCLE TO EXPRESS THE ANGLE IN RADIANS.



1. TRIANGLE A IS SIMILAR TO TRIANGLE B BECAUSE OF THEOREM #1 (AA).



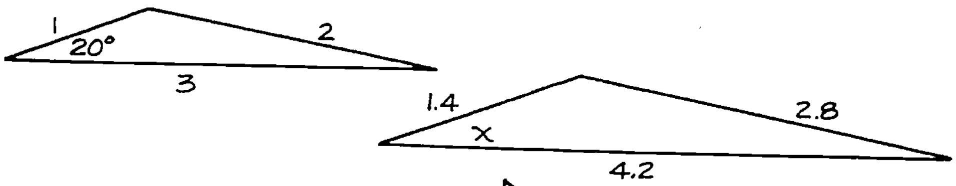
2. TRIANGLE A IS SIMILAR TO TRIANGLE B BECAUSE OF THEOREM #2 (SAS).



3. TRIANGLE A IS SIMILAR TO TRIANGLE B BECAUSE OF THEOREM #3 (SSS).

FOR EACH OF THE FOLLOWING TRIANGLES STATE THE THEOREM THAT INSURES SIMILARITY AND DETERMINE x.

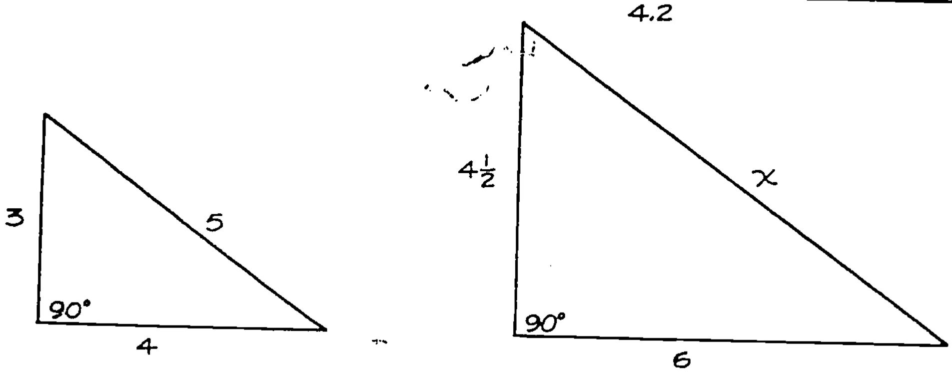
1.



THEOREM #3 (SSS)

$x = 20°$

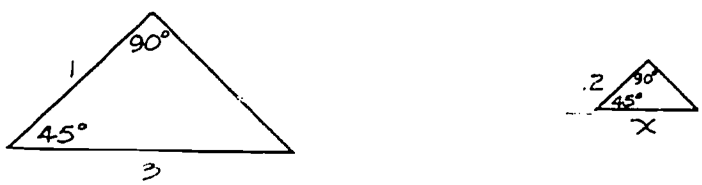
2.



THEOREM #2 (SAS)

$x = 7\frac{1}{2}$

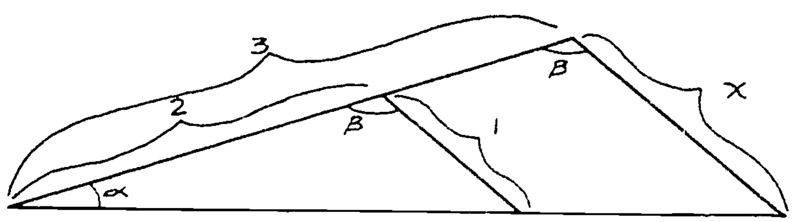
3.



THEOREM #1 (AA)

$x = .6$

4.

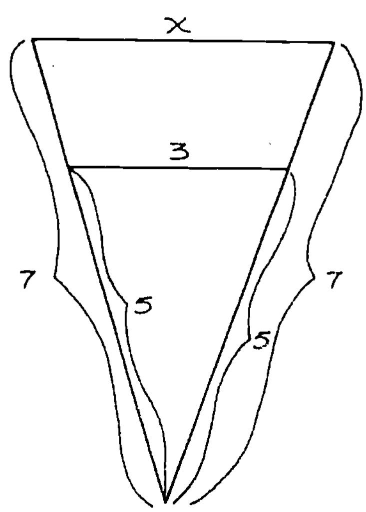


THEOREM #1 (AA)

$\frac{3}{2} = \frac{x}{1}$

$x = 1\frac{1}{2}$

5.

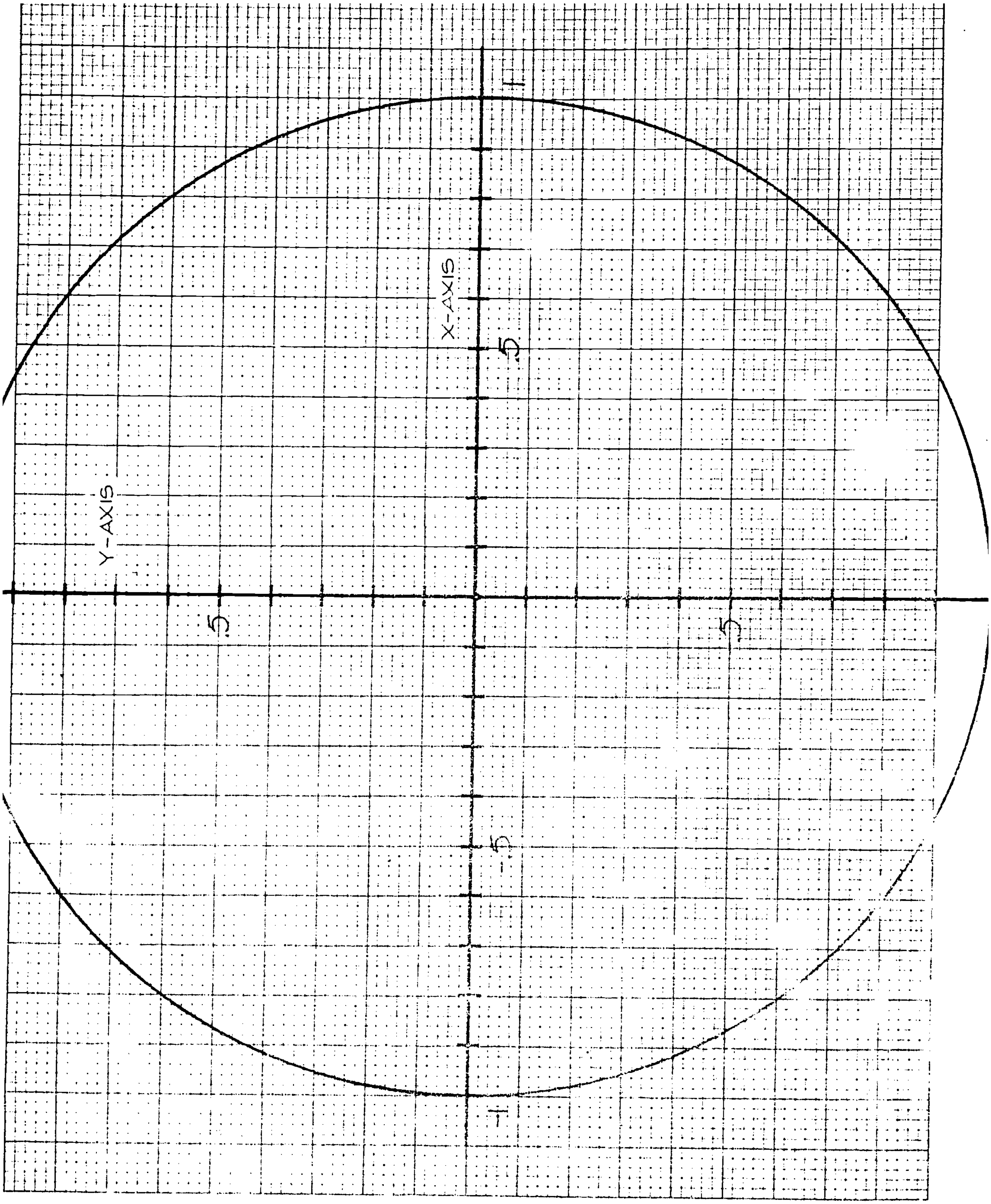


THEOREM #2 (SAS)

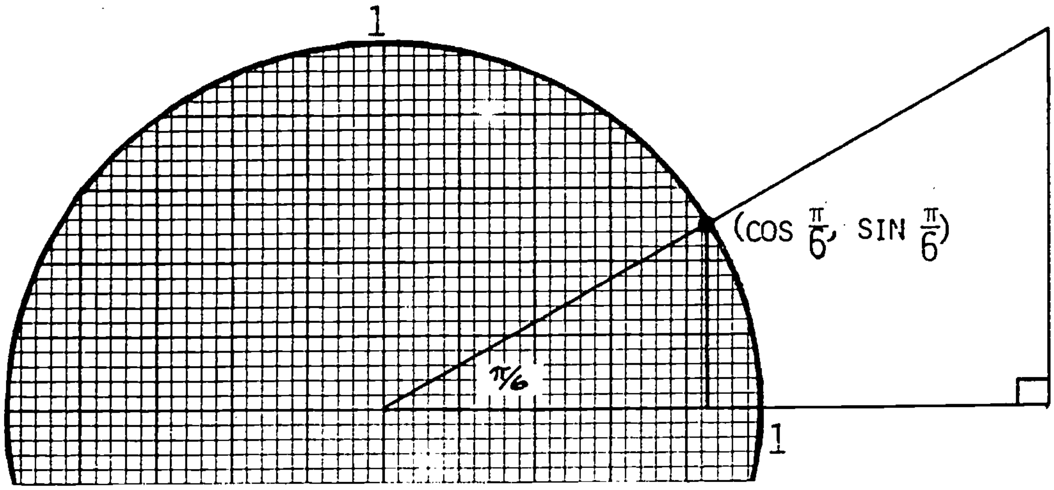
$x = 4\frac{1}{5}$

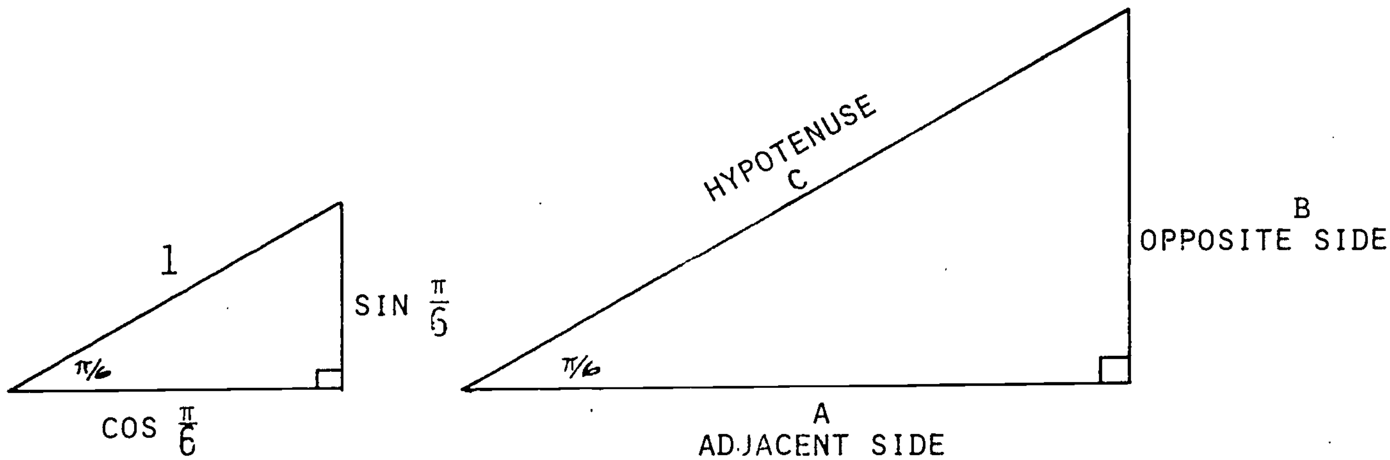
BRIEF TABLE OF TRIG FUNCTIONS

θ	COS θ	SIN θ
0°		
30°		
45°		
90°		
150°		
180°		
210°		
270°		
300°		
360°		
390°		
540°		
720°		



Transparency Master IV-M-17b





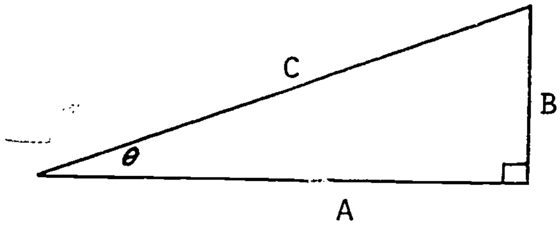
1. THE TWO TRIANGLES ARE SIMILAR. WHY?

2. $$\frac{\cos\frac{\pi}{6}\text{ RAD}}{1} = \frac{A}{C} \quad \text{WHY?}$$

$$\cos\frac{\pi}{6}\text{ RAD} = \frac{\text{ADJACENT SIDE}}{\text{HYPOTENUSE}} = \frac{A}{C} \simeq .87$$

3. $$\frac{\sin\frac{\pi}{6}\text{ RAD}}{1} = \frac{B}{C} \quad \text{WHY?}$$

$$\sin\frac{\pi}{6}\text{ RAD} = \frac{\text{OPPOSITE SIDE}}{\text{HYPOTENUSE}} = \frac{B}{C} \simeq .5$$



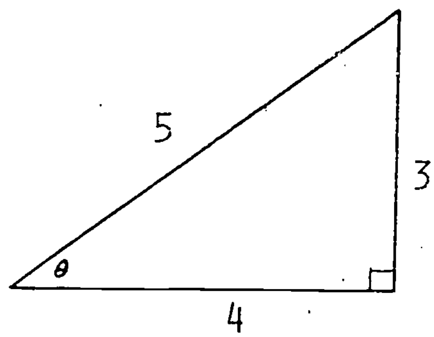
$$\cos\theta = \frac{A}{C} = \frac{\text{ADJACENT SIDE}}{\text{HYPOTENUSE}}$$

$$\sin\theta = \frac{B}{C} = \frac{\text{OPPOSITE SIDE}}{\text{HYPOTENUSE}}$$

<u>DEFINITION</u>: THE TANGENT FUNCTION IS DEFINED AS FOLLOWS:

$$\boxed{\tan\theta = \frac{B}{A} = \frac{\text{OPPOSITE SIDE}}{\text{ADJACENT SIDE}}}$$

<u>EXAMPLE</u>:

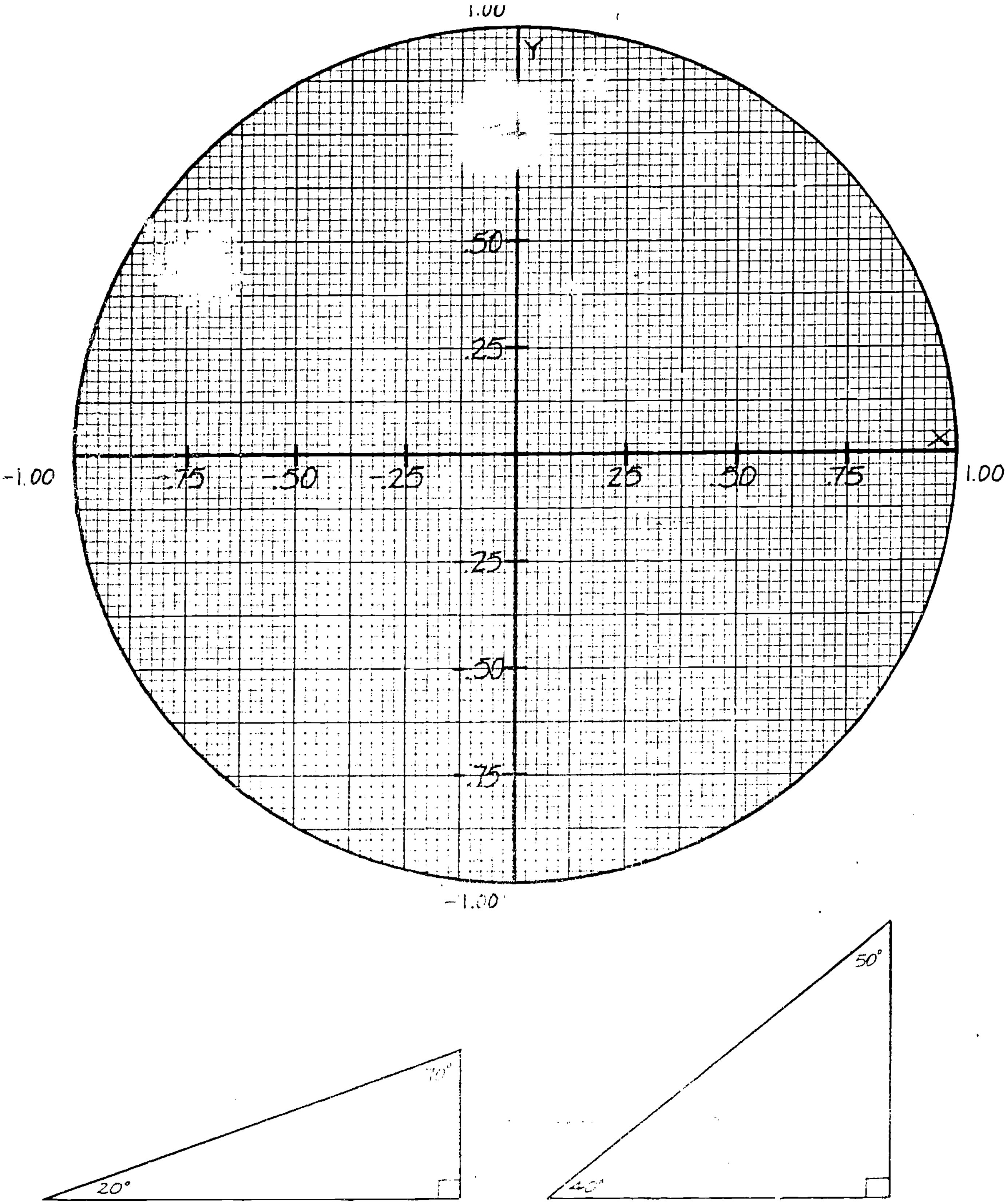


ANSWERS:

$$\cos\theta = \frac{\text{ADJACENT SIDE}}{\text{HYPOTENUSE}} = \qquad \frac{4}{5}$$

$$\sin\theta = \frac{\text{OPPOSITE SIDE}}{\text{HYPOTENUSE}} = \qquad \frac{3}{5}$$

$$\tan\theta = \frac{\text{OPPOSITE SIDE}}{\text{ADJACENT SIDE}} = \qquad \frac{3}{4}$$



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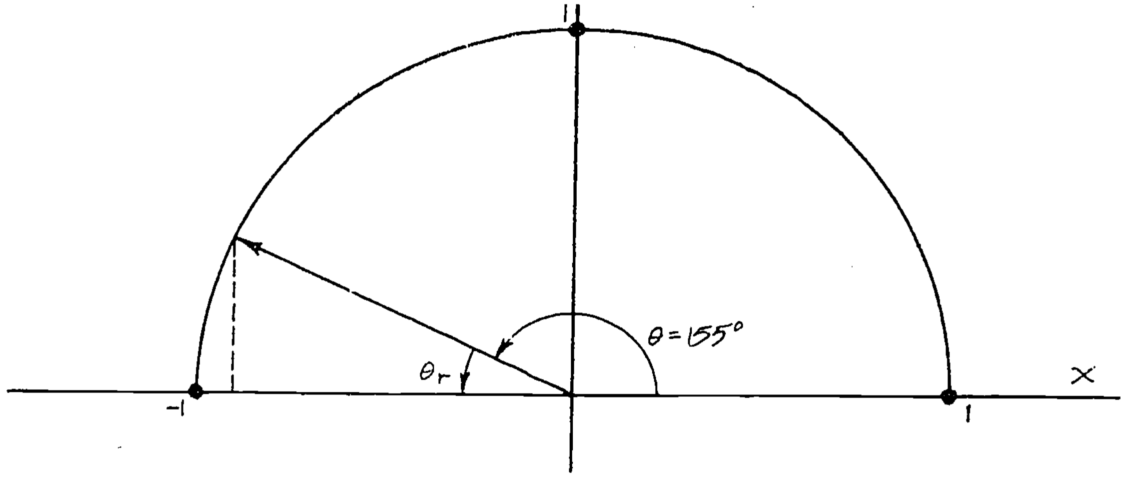
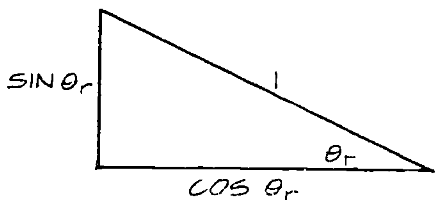
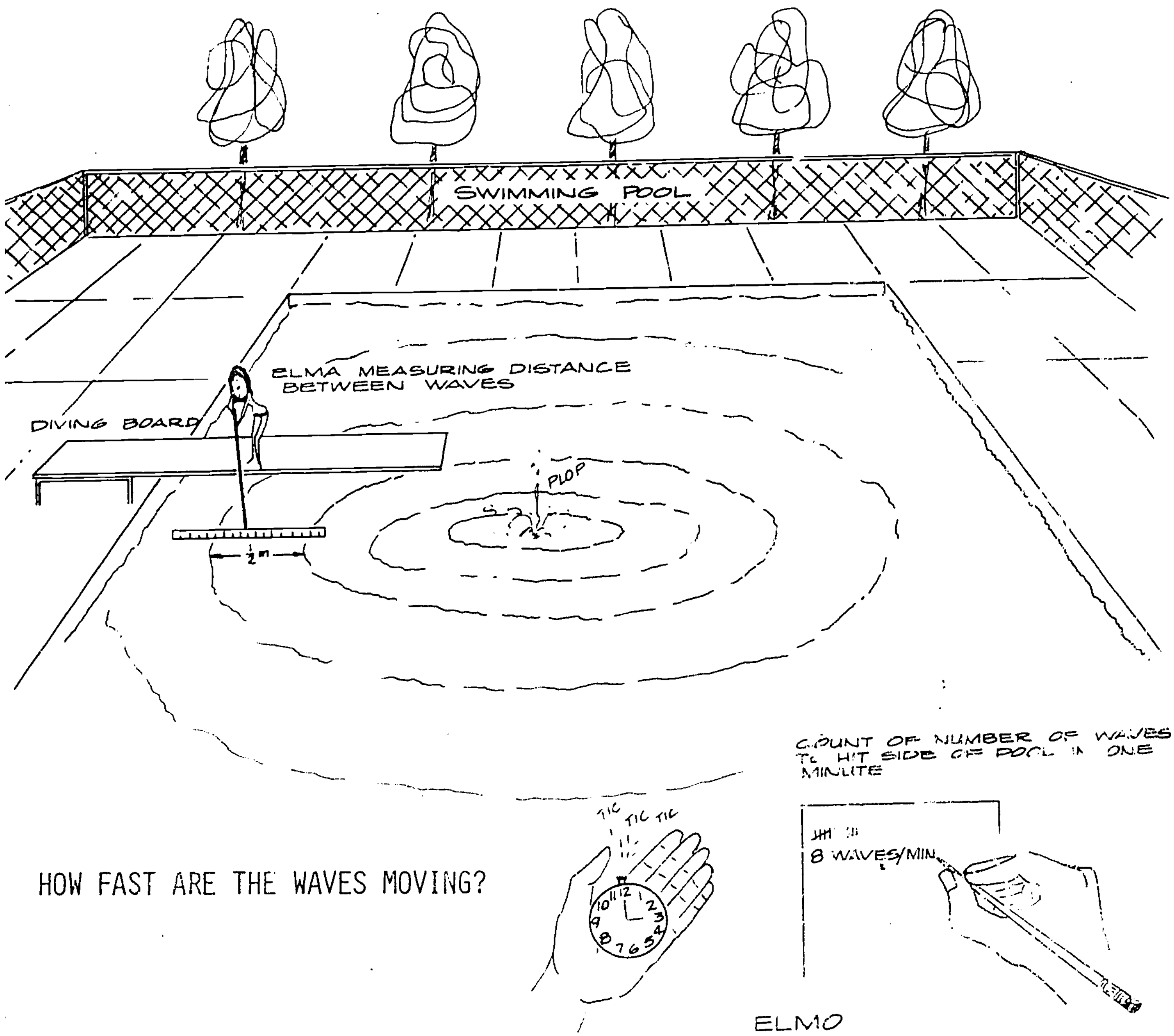
Transparency IV-M-20a



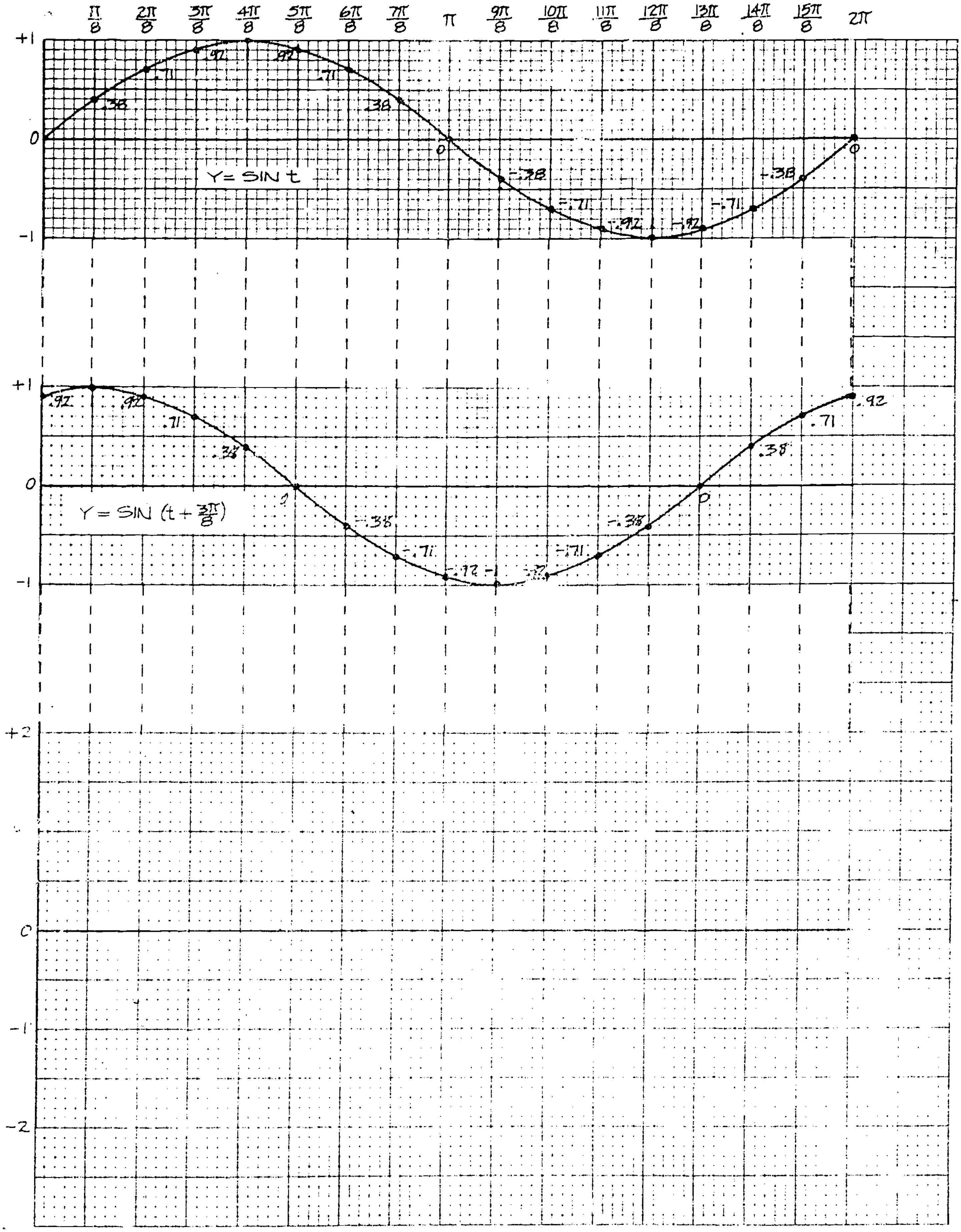


EXAMPLE 1: FIND cos 155° AND SIN 155°.

1.	θ_r =	1. 25°
2.	FROM THE TABLE, cos θ_r =	2. .906
3.	SIN θ_r =	3. .423
4.	THE ANGLE 155° LIES IN WHICH QUADRANT?	4. II
5.	THE SIGN OF cos 155° MUST BE _____?	5. NEGATIVE
6.	THE SIGN OF SIN 155° MUST BE _____?	6. POSITIVE
7.	cos 155° =	7. -.906
8.	SIN 155° =	8. .423



HOW FAST ARE THE WAVES MOVING?



Transparency Master IV-M-26

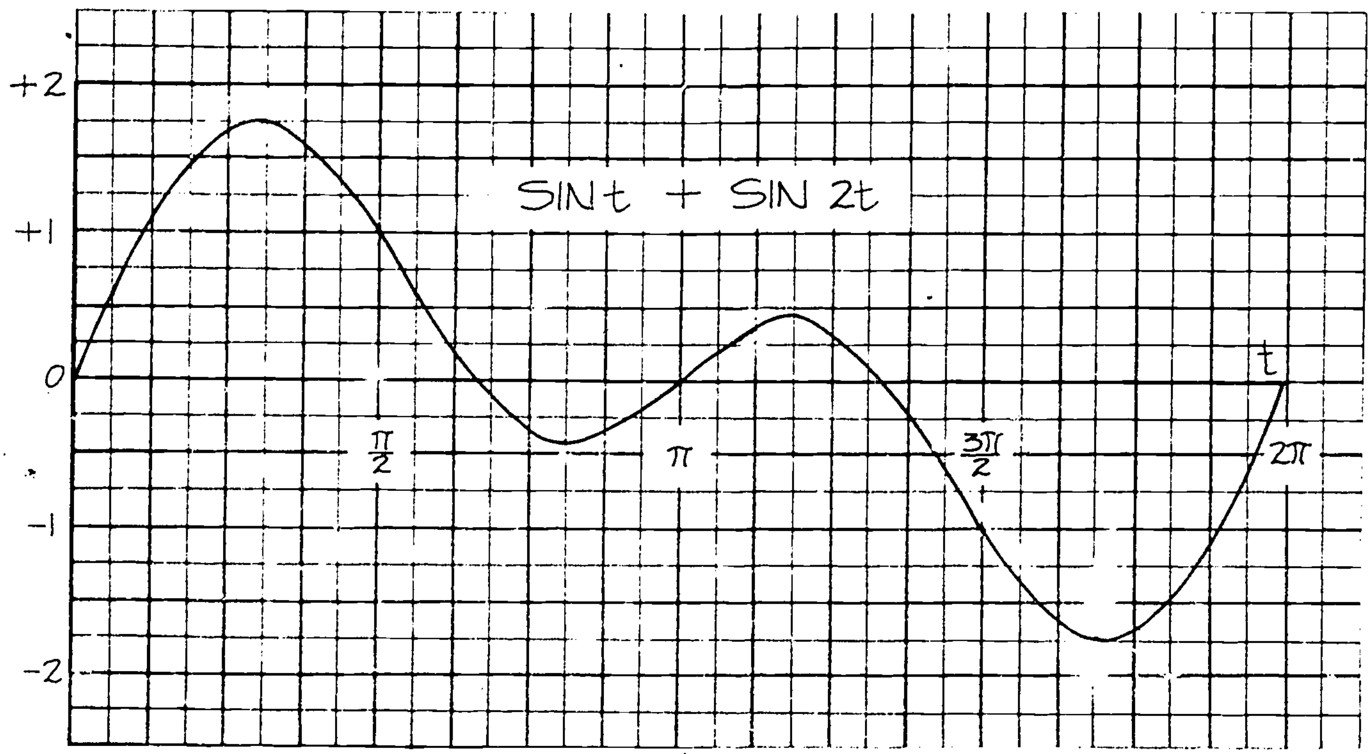


FIGURE 1

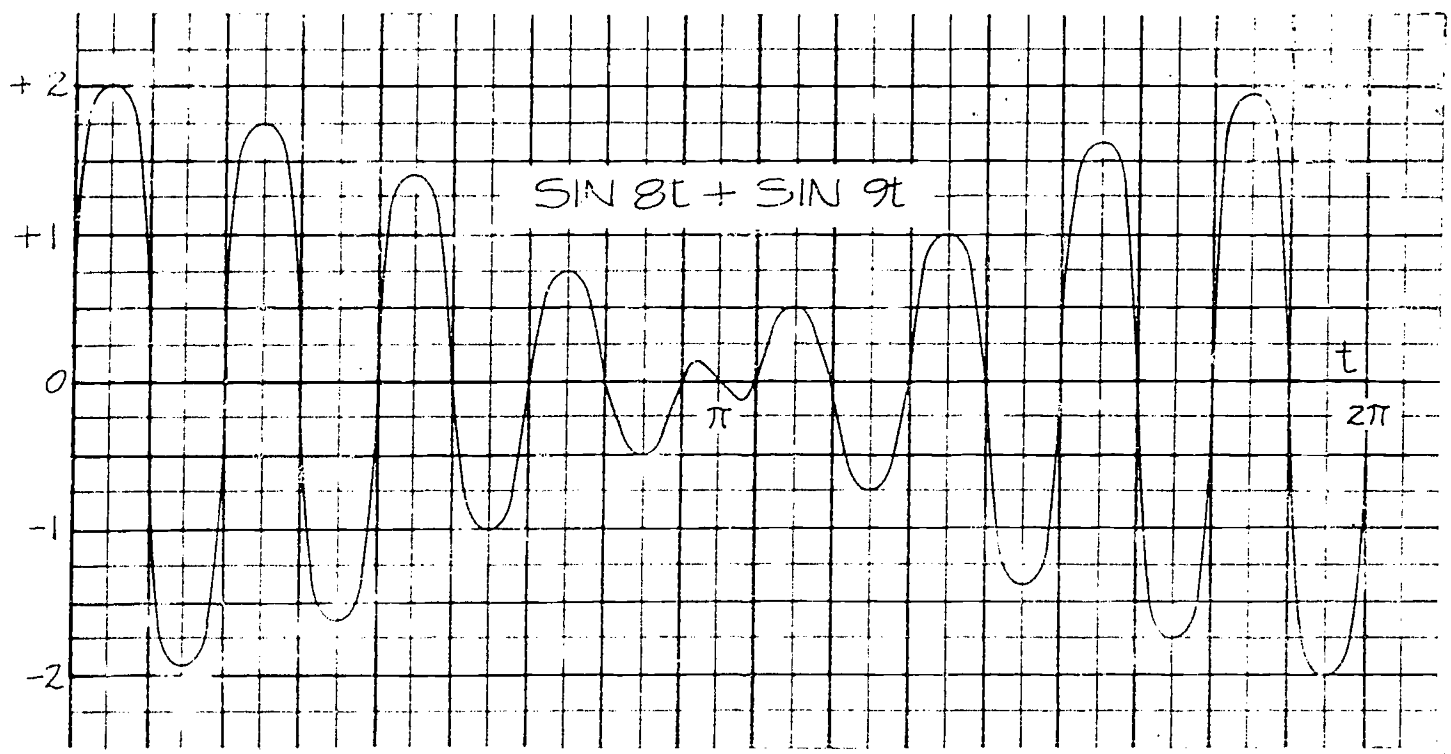
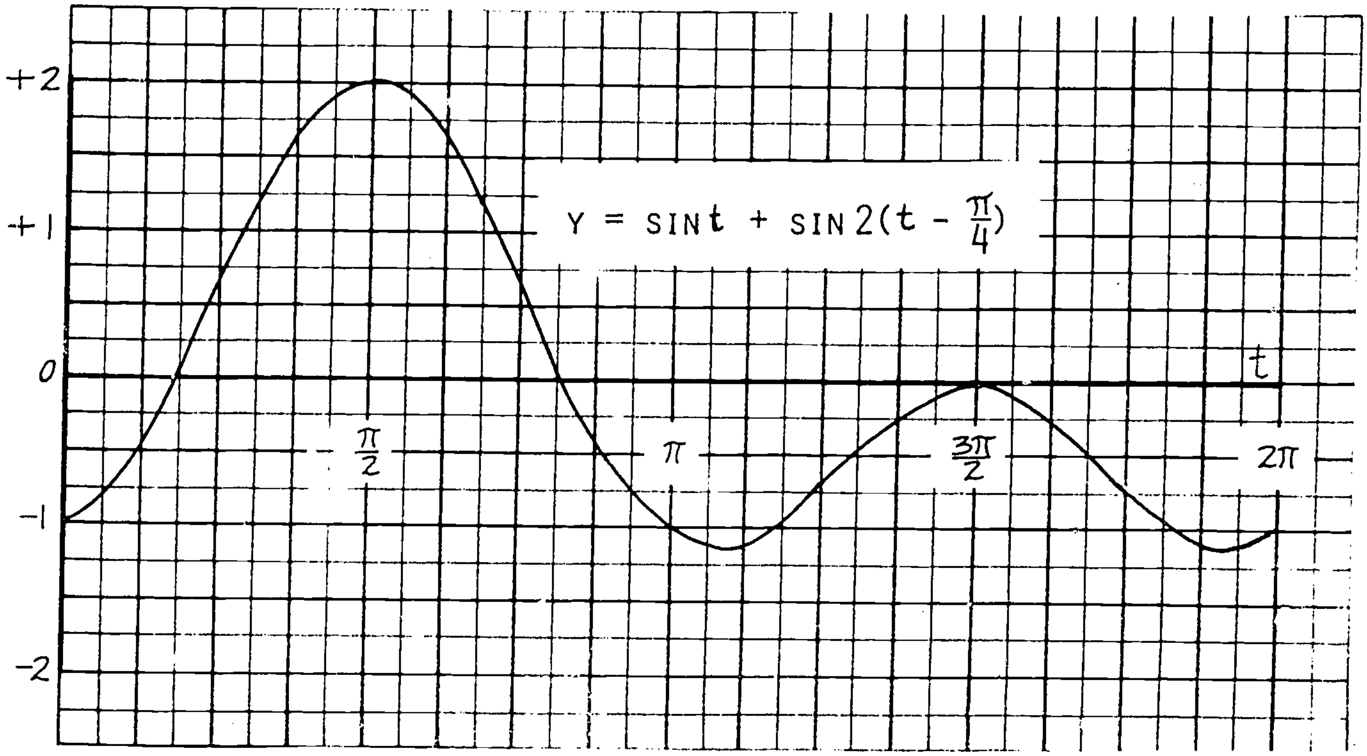
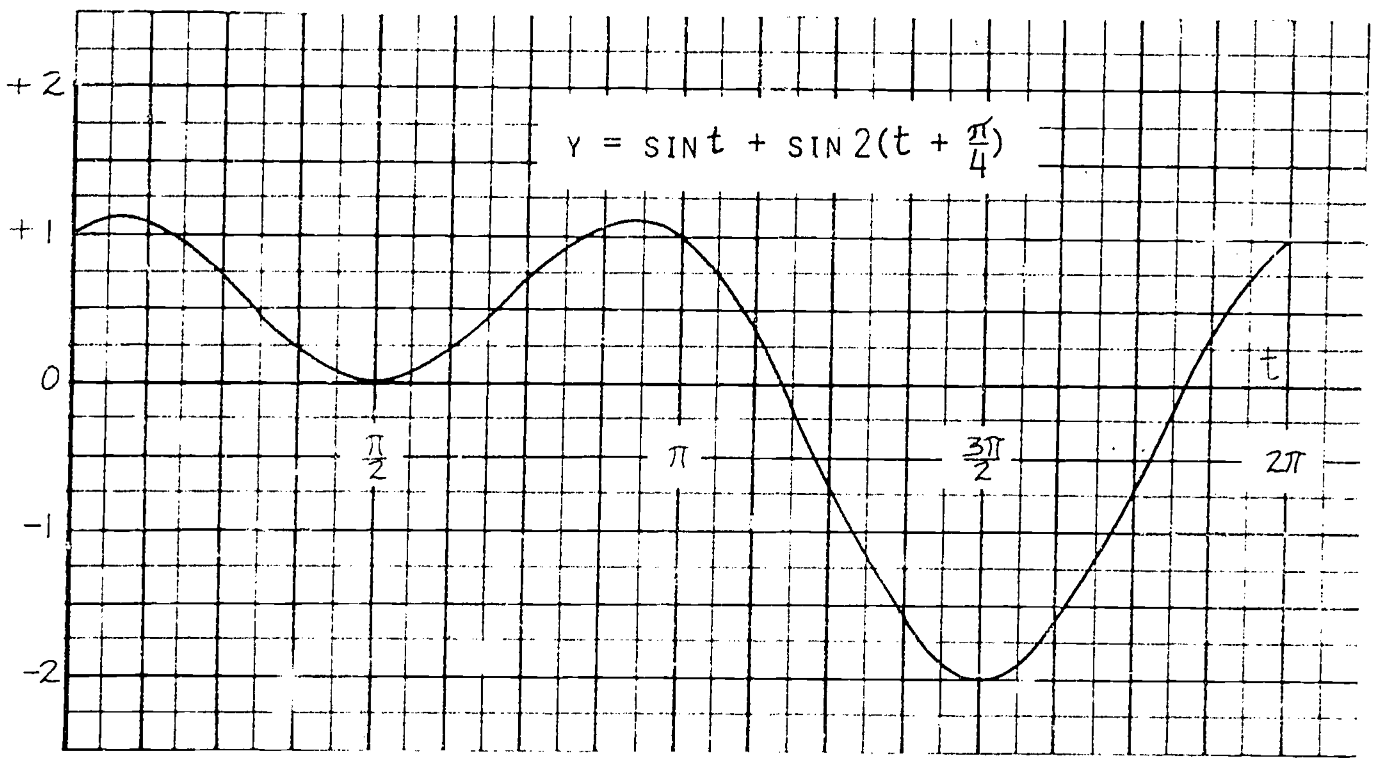


FIGURE 2





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Transparency Master IV-M-29b

SAMPLE CALCULATION: THE MEAN

$$\boxed{\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}}$$

DATA: $x_1 = 1.40$ $x_3 = 1.35$

$x_2 = 1.31$ $x_4 = 1.30$

$$\bar{x} = \frac{\sum_{i=1}^{4} x_i}{4}$$

$$= \frac{x_1 + x_2 + x_3 + x_4}{4}$$

$$= \frac{1.40 + 1.31 + 1.35 + 1.30}{4}$$

$$= \frac{5.36}{4}$$

$$= 1.34$$

THE MEAN VALUE, $\bar{X}$, IS 1.34

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Transparency Master IV-M-X3a

SAMPLE CALCULATION: MEAN DEVIATION

$$\text{MEAN DEVIATION} = \frac{\sum_{i=1}^{n} |x_i - \bar{x}|}{n}$$

DATA:

$x_1 = 1.40$ $\quad x_3 = 1.35$ $\quad \bar{x} = 1.34$

$x_2 = 1.31$ $\quad x_4 = 1.30$

$$|x_1 - \bar{x}| = .06$$

$$|x_2 - \bar{x}| = .03$$

$$|x_3 - \bar{x}| = .01$$

$$|x_4 - \bar{x}| = .04$$

$$\text{MEAN DEVIATION} = \frac{\sum_{i=1}^{4} |x_i - \bar{x}|}{4}$$

$$= \frac{|x_1 - \bar{x}| + |x_2 - \bar{x}| + |x_3 - \bar{x}| + |x_4 - \bar{x}|}{4}$$

$$= \frac{.14}{4}$$

$$= .035$$

Transparency Master IV-M-X3b

SAMPLE CALCULATION: STANDARD DEVIATION

$$s^2 = \frac{1}{n-1}\left[\sum_{i=1}^{n}(x_i - \bar{x})^2\right]$$

$$\bar{x} = 1.34$$

	x_i	$x_i - \bar{x}$	$(x_i - \bar{x})^2$
x_1	1.40	.06	.0036
x_2	1.31	-.03	.0009
x_3	1.35	.01	.0001
x_4	1.30	-.04	.0016
		sum:	$.0062 = \sum_{i=1}^{4}(x_i - 1.34)^2$

CALCULATION OF VARIANCE, s^2

$$s^2 = \frac{1}{n-1}\left[\sum_{i=1}^{n}(x_i - \bar{x})^2\right], \quad n = 4$$

$$s^2 = \frac{1}{4-1}\left[\sum_{i=1}^{4}(x_i - 1.34)^2\right]$$

$$s^2 = \frac{1}{3}(.0062)$$

$$s^2 \simeq .0021$$

CALCULATION OF STANDARD DEVIATION, s

$$s = \sqrt{s^2}$$

$$s \simeq \sqrt{.0021}$$

$$s \simeq \sqrt{21 \times 10^{-4}}$$

$$s \simeq 4.6 \times 10^{-2}$$

SAMPLE CALCULATION: WEIGHTED MEAN

$$\bar{x}_w = \frac{\sum_{i=1}^{n} w_i \bar{x}_i}{\sum_{i=1}^{n} w_i}$$

$$w_i = \frac{1}{s_i}$$

DATA:

$\bar{x}_1 = 1.35$ $\quad s_1 = .02$ $\quad w_1 = \frac{1}{s_1} = 50$

$\bar{x}_2 = 1.30$ $\quad s_2 = .04$ $\quad w_2 = \frac{1}{s_2} = 25$

$$\bar{x}_w = \frac{w_1\bar{x}_1 + w_2\bar{x}_2}{w_1 + w_2}$$

$$= \frac{(50 \cdot 1.35) + (25 \cdot 1.30)}{50 + 25}$$

$$= \frac{67.5 + 32.5}{75}$$

$$\simeq 1.33$$

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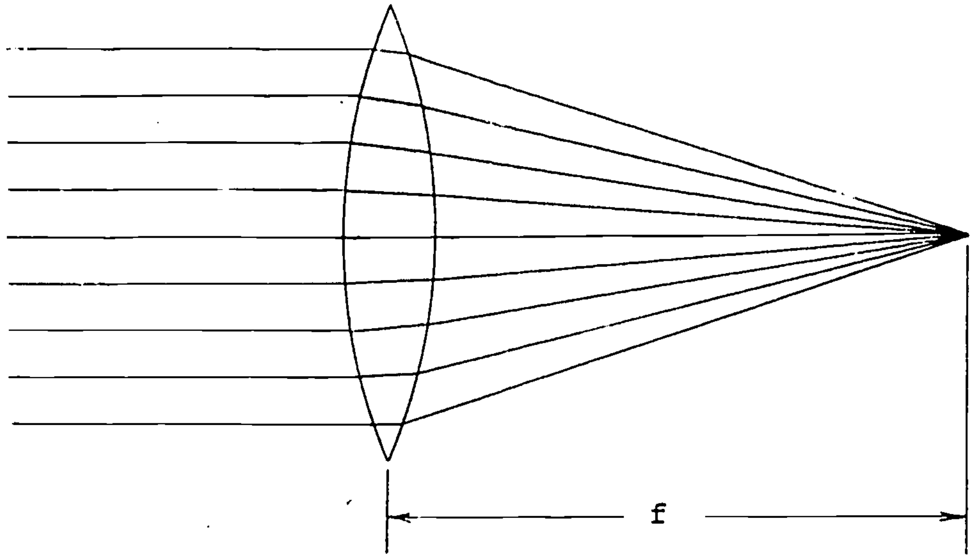
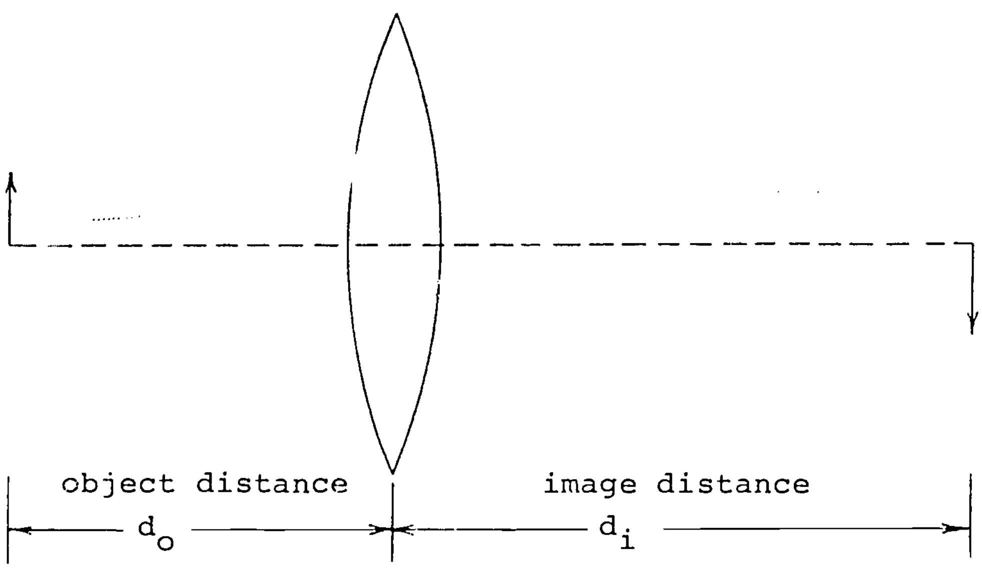


$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$



Transparency Master IV-M-Y1a

SAMPLE CALCULATIONS: FOCAL LENGTH

$$\boxed{\begin{gathered}\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} \\[1em] f = \frac{d_o d_i}{d_o + d_i}\end{gathered}}$$

DATA: $d_o = 20\ \text{cm}$

$d_i = 30\ \text{cm}$

$$f = \frac{(20\ \text{cm}) \cdot (30\ \text{cm})}{20\ \text{cm} + 30\ \text{cm}}$$

$$= \frac{600\ \text{cm}^2}{50\ \text{cm}}$$

$$= 12\ \text{cm}$$

THE FOCAL LENGTH IS 12 CM.

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SAMPLE CALCULATION: STANDARD DEVIATION

	f_1	f_2	f_3
	10.0 cm	10.5 cm	9.8 cm
$\bar{f}$ (cm)	10.1		
$\bar{f}$ (cm)	-.1	.4	-.3
$(f_i - \bar{f})^2$ (cm^2)	.01	.16	.09
s (cm)	.36		

$$s = \sqrt{\frac{\sum_{i=1}^{3} (f_i - \bar{f})^2}{2}}$$

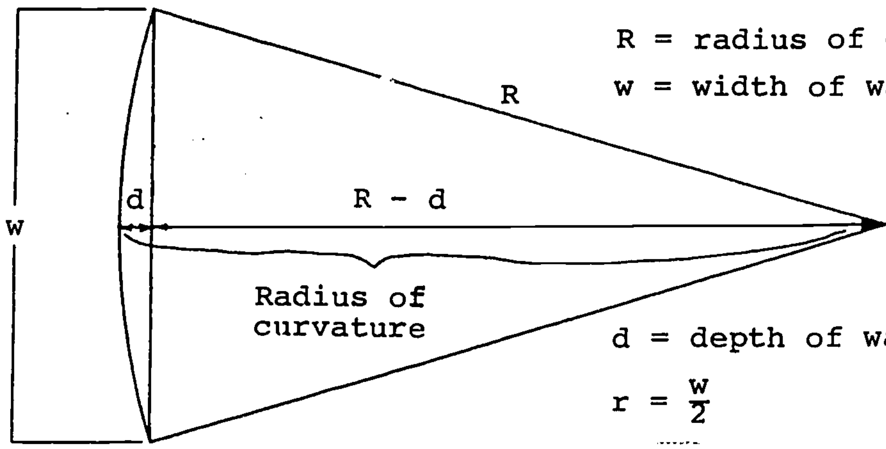
$$= \sqrt{\frac{.01 + .16 + .09}{2}}$$

$$= \sqrt{.13}$$

$$= \sqrt{13 \times 10^{-2}}$$

$$= \sqrt{13} \times 10^{-1}$$

$$\simeq .36$$



R = radius of curvature

w = width of watch glass

d = depth of watch glass

$r = \frac{w}{2}$

FROM THE PYTHAGOREAN THEOREM,

$$R^2 = r^2 + (R - d)^2$$

$$R^2 = r^2 + R^2 - 2Rd + d^2$$

$$2Rd = r^2 + d^2$$

$$\boxed{R = \frac{r^2 + d^2}{2d}}$$

$$\boxed{f_\theta = \frac{R}{2(n-1)}}$$

0% sucrose: $n = 1.333$

25% sucrose: $n = 1.372$

50% sucrose: $n = 1.420$

Transparency Master IV-M-Y2b

DATA FROM ORINDA STUDY

TEST GROUP: 70 STUDENTS (140 EYES) IN THE AGE RANGE OF 8, 9 AND 10 YEARS OF AGE.

ACUITY RATIO	20/5	20/7.5	20/10	20/15	20/20	20/25	20/40	20/50	20/100	LESS THAN 20/100
SCALE	1	2	3	4	5	6	7	8	9	10
NUMBER OF EYES	0	0	0	0	89	37	5	0	7	2

MEAN - 5.61

MEDIAN - 5

STANDARD DEVIATION - 1.1

DESCRIPTION OF SAMPLE: "ORINDA IS A RESIDENTIAL SUBURBAN COMMUNITY EAST OF OAKLAND AND BERKELEY, CALIFORNIA. THE ORINDA UNION SCHOOL DISTRICT COMPRISES FIVE SCHOOLS. THE POPULATION, 99 PER CENT CAUCASIAN, LIVES IN ABOVE-AVERAGE, SINGLE-FAMILY DWELLINGS. OCCUPATIONAL GROUPS, IN ORDER OF SIZE, ARE: PROFESSIONAL AND WHITE COLLAR (IN MANAGERIAL AND SUPERVISORY CAPACITY--90 PER CENT), SERVICE AND CLERICAL PERSONNEL, SKILLED OR SEMISKILLED LABOR, AND UNSKILLED AND FARM LABOR. ORINDA IS CONSIDERED TO HAVE THE HIGHEST PER CAPITA INCOME IN CONTRA COSTA COUNTY."*

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*THE ORINDA STUDY